Bottom-up EFTs and model independence: Some historical roots

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Max Planck Institute for the History of Science (MPIWG)

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Bottom-up Effective Field Theories (EFTs) have taken an increasingly central place since the early 1980s, esp. in areas where we have little information about new physics or the non-perturbative structure of well-known interactions.

Baryon- and Lepton-Nonconserving Processes

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(Received 13 August 1979)

ON THE LOW ENERGY STRUCTURE OF QCD☆

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Received 21 March 1983

EFFECTIVE LAGRANGIAN ANALYSIS OF NEW INTERACTIONS AND FLAVOUR CONSERVATION

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Received 2 October 1985

EFFECTIVE LAGRANGIANS FOR BOUND STATE PROBLEMS IN QED, QCD, AND OTHER FIELD THEORIES

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Received 13 August 1985

General relativity as an effective field theory: The leading quantum corrections

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(Received 26 May 1994)

This has especially been the case in Beyond the Standard Model (BSM) particle physics for the last 10-15 years, with the development of new techniques for classifying and analyzing higher-order operators in EFT expansions.

Dimension-six terms in the Standard Model Lagrangian[1]

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Hilbert series for constructing Lagrangians: Expanding the phenomenologist's toolbox

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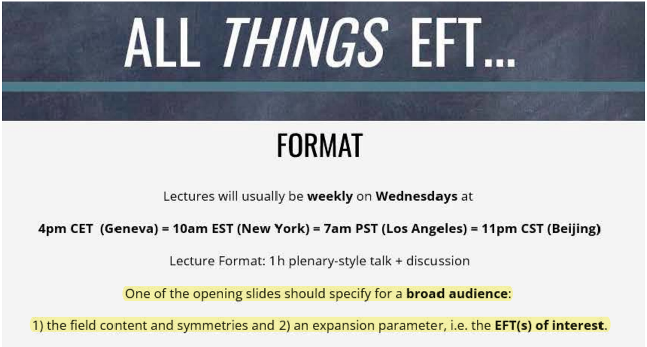
(Received 5 April 2015; published 19 May 2015)

- Part of a broader turn to experimental and theoretical model-independent research strategies in BSM particle physics (Bechtle et al., 2022).

Today: Look at the early historical development of bottom-up EFTs and model-independent research strategies.

Bottom-up EFT: Systematic field-theoretic expansion in some ratio(s) of scales governed by a set of principles (e.g., symmetries), which typically takes the form of a local polynomial expansion in field variables and their derivatives.



- To be contrasted with "top-down EFTs" for which there is an explicit derivational relation to some high-energy (effective) theory.

Warning: The history of bottom-up EFTs is quite complicated and involves a variety of traditions, including:

1) A more phenomenological tradition going back to Fermi's theory of β-decays in 1934 and which gained a new theoretical texture in the early 1950s when it was realized that Fermi-type models are not renormalizable (in the power-counting sense);

2) A more mathematical tradition going back to Wilson's operator product expansion in 1969 and which quickly found phenomenological applications with the short-distance behavior of hadrons when combined with Renormalization Group (RG) methods in the 1970s;

3) A mixed tradition going back to Weinberg's, Schwinger's, and many others' works on phenomenological Lagrangians in the late 1960s.

Focus: I will look more specifically at the phenomenological Lagrangian tradition through the lens of Steven Weinberg's works on low-energy pion-nucleon physics and examine what led him to formulate a first prototype of bottom-up EFT in 1966-67.

Claims:

- Despite a very rudimentary power-counting scheme, Weinberg's mature concept of a bottom-up EFT as a systematic local field-theoretic expansion in some scale is already pretty much in place at that time;
- Understanding the distinctive features of this prototype requires going much before his first works on current algebra.

Outline:

1. Some key elements in Weinberg's trajectory before 1966
2. Weinberg's first prototype of bottom-up EFT in 1966-67
3. Conclusion: Model independence in 1967?

1. Some key elements in Weinberg's trajectory before 1966

The story begins at Princeton, where Weinberg did his PhD under the supervision of Sam Treiman between 1954-57 on the application of renormalization theory to the effects of strong and electromagnetic interactions in weak decays.

THE RÔLE OF STRONG INTERACTIONS IN DECAY PROCESSES

By

Steven Weinberg

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Central issue: Weak decay processes appear to be much more complicated than original expected in the 1930s (e.g., $n \rightarrow p + e^- + \bar{\nu}_e$) and may involve, in particular, higher-order electromagnetic and strong interactions (e.g., $N \rightarrow N'' + \pi \rightarrow N''' + e + \nu + \pi \rightarrow N' + e + \nu$).

Question: Any principle to identify "primary" interaction terms and reduce the set of possible "decay mechanisms"?

If we take QFT seriously, the principle of renormalizability leads us to select interaction terms that:

(i) Generate a limited number of different types of UV divergences;

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(i) Generate a limited number of different types of UV divergences;

(ii) Cancel each others' UV-divergent contributions.

- Weinberg's approach had important limitations (e.g., restriction to first order contributions in the weak interaction coupling constants).

Still, this early work sets the tone for the sequel: Weinberg was very much concerned from the get-go with finding systematic principles to organize (low-energy) particle phenomena.

CERN (1979)

From the beginning it seemed to me to be a wonderful thing that very few quantum field theories are renormalizable. Limitations of this sort are, after all, what we most want; not mathematical methods which can make sense out of an infinite variety of physically irrelevant theories, but methods which carry constraints, because these constraints may point the way toward the one true theory. (Weinberg, 1980, p. 517)

Once his dissertation defended, Weinberg moved to Columbia in 1957 and Berkeley in 1959, where he became assistant professor in 1961.

A central trend during those years: Renormalizability will not be sufficient as a heuristic guide to reduce the number of potentially relevant theories; we will also need symmetries.

Charge Symmetry of Weak Interactions*

STEVEN WEINBERG
Columbia University, New York, New York
(Received June 25, 1958)

On the Phase Factors in Inversions (*).

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S. WEINBERG (*.*)
Columbia University - New York, N.Y.

(ricevuto il 21 Luglio 1959)

- A concern especially spurred by the discovery of parity violation in weak decays and the subsequent works of Marshak & Sudarshan, Feynman & Gell-Mann, and Sakurai on V-A interactions in 1957-58.

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To make the matter even more confusing: Clear that many symmetries governing (low-energy) phenomena were not going to be exact.

- Do "approximate symmetries" reflect some "approximate order" in the world? And why using exact symmetries if you directly have to break them by hand?

Crucial input: Clear examples from Nambu and Jona-Lasinio (1961a,b) and Goldstone (1961) where one could account for qualitative differences between particles without breaking the symmetries of a theory.

Sterling Hall, University of Wisconsin

I think that week with Jeffrey Goldstone in Madison was really the first time I began to think about these things seriously. [...] I fell in love with symmetry breaking. (Weinberg, Interview with Crease and Mann, November 28, 1984)

- Spontaneous symmetry breaking didn't really account for approximate symmetries. But it still cleared the path in the search for symmetries without losing the tie to particle phenomenology.

... especially in the realm of the strong interaction:

> *[...] I would say that the greatest triumph of the Goldstone theorem is that it gives a 'raison d'être' for the pion as an almost massless particle. From this point of view, it is not important whether the Goldstone theorem has been rigorously proved; the important thing is that it tells us how the strong interactions could keep the pion mass so small. (Weinberg, October 1967, 14th Solvay Conference on Physics)*

Still, most physicists were largely convinced in the early 1960s that relativistic QFT models were ineffective in this context (i.e., perturbative renormalization methods unreliable for large couplings).

The S-matrix approach was in vogue at Berkeley. Yet Weinberg had some reservations about it (esp. with respect to electromagnetic and gravitational interactions).

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Weinberg's new program (1963-65): Systematic derivation of multi-particle scattering amplitudes from a set of first principles (incl. Lorentz invariance, quantum theory, causality condition).

- In particular: Show that it is possible to recover most features of photons and gravitons without assuming anything about their dynamics (Weinberg, 1964a,b,c, 1965).

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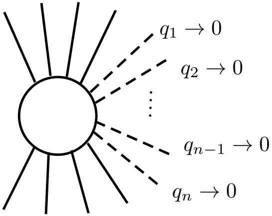
4) New conception of a Lagrangian or Hamiltonian field-theoretic model as the most general local functional of field variables and their derivatives compatible with a set of principles (e.g., Lorentz invariance);

5) Systematic scheme for deriving arbitrary soft particle amplitudes.

Having dealt with photons and gravitons, Weinberg tried to see whether the same could be done with soft pions in 1965-66.



At the time, the general method for dealing with one or two soft pions involved a mixture of the partially conserved axial current (PCAC) and current algebra hypotheses (cf., Nambu and Lurié, 1962; Adler, 1965).

1) **PCAC hypothesis (1960)**: The divergence of the axial vector current component of weak interactions $\partial_\mu A^\mu_a$ accounts sufficiently well for the creation and annihilation of approximately massless pions π_a.

- In practice: Use $\partial_\mu A^\mu_a = f_\pi \pi_a$ in the soft pion limit to compute matrix elements (f_π pion decay constant).

2) **Current algebra hypothesis (1964)**: The physical currents involved in the electromagnetic and weak decays of strongly interacting particles satisfy definite algebraic relations reflecting the symmetry properties of these particles (e.g., $SU(3)$ for mesons and baryons).

- In practice: Take the currents $A^\mu_a(x)$, $V^\mu_b(x)$, etc. as primary variables and use their algebraic relations to obtain constraints on scattering amplitudes and derive empirical relations between key quantities.

Generalization of the following simple case (Weinberg, 1966a,b):

1) Start with the soft-pion matrix element for $N + \pi \to N + \pi$:

$$M_{ab} = \int d^4x d^4y e^{iqx} e^{-iky} \langle N | T\{\pi_a(x)\pi_b(y)\} | N \rangle;$$

2) Replace π_a by $\frac{1}{f_\pi}\partial_\mu A^\mu_a$ (PCAC);

3) Bring out the derivatives to obtain terms such as $\partial_\mu \partial_\nu T\{A^\mu_a A^\nu_b\}$ and $[A^\mu_a, A^\nu_b]$ and use current algebra commutators;

4) Keep only the leading order terms in the soft momentum limit $(q, k \to 0)$ and obtain a simplified relation between M_{ab} and a sum of matrix elements such as $\langle N | A^\mu_a | N \rangle$ and $\langle N | V^\mu_a | N \rangle$.

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2) In general, we need a particular Lagrangian model to compute $[\partial_\mu A_a^\mu, A_b^\nu]$;

3) Method not very fruitful beyond the soft-pion limit since current algebra hides the underlying dynamical details of soft-particle processes.

Weinberg moved to Harvard in 1966 as Loeb Lecturer, and became visiting professor at MIT in 1967 and full professor in 1969.

- First prototype of bottom-up EFT in 1966-1967 behind his seminal 1979 article on phenomenological Lagrangians.

DYNAMICAL APPROACH TO CURRENT ALGEBRA

Steven Weinberg*
Department of Physics, University of California, Berkeley, California
(Received 12 December 1966)

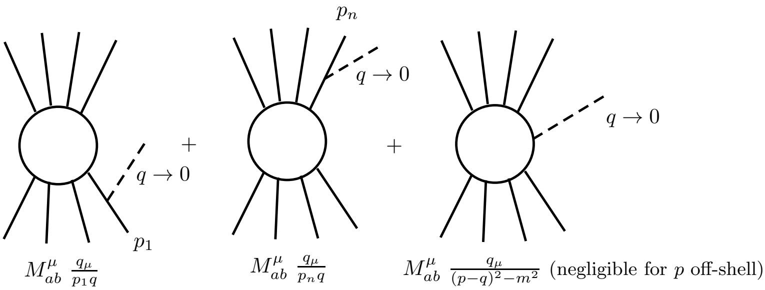
An effective Lagrangian for soft-pion interactions is constructed such that lowest order perturbation theory precisely reproduces the results of current algebra.

- **Basic idea**: Use a Lagrangian field theory to re-derive systematically the results of PCAC & current algebra for N soft pions.

2. Weinberg's first prototype of bottom-up EFT in 1966-67

Starting point: Any model satisfying PCAC & current algebra gives the same results for the emission and absorption of an arbitrary number of soft pions (if we appropriately adjust its free parameters).

However, some models are better than others. In particular, models involving only gradient couplings (e.g., $\bar{N}\partial_\mu \pi \gamma^\mu N$) yield amplitudes with soft pions coming out only of external lines at lowest order.



Weinberg started with the (linear) σ-model in 1967:

$$\mathcal{L}_\sigma = -\bar{N}(\gamma^\mu\partial_\mu + m_N)N - \frac{1}{2}(\partial_\mu\vec{\pi}.\partial^\mu\vec{\pi} + m_\pi^2\vec{\pi}^2) - \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + m_\sigma^2\sigma^2)$$

$$- g_1(\vec{\pi}^2 + \sigma^2)\sigma - g_2(\vec{\pi}^2 + \sigma^2)^2$$

$$- \; g_3\bar{N}(\sigma + i\vec{\tau}.\vec{\pi}\gamma_5)N$$

(N: nucleon field; $\vec{\pi}$: pion field; σ: scalar field; $m_{N,\pi,\sigma}$: their respective mass; g_i: some couplings.)

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(N: nucleon field; $\vec{\pi}$: pion field; σ: scalar field; $m_{N,\pi,\sigma}$: their respective mass; g_i: some couplings.)

Issue: This model is not well-suited to re-derive easily the soft-pion amplitudes obtained with PCAC & current algebra.

Weinberg proposes to transform $\mathcal{L}_\sigma$ by redefining the nucleon field N through the following non-linear chiral transformation:

$$N = \frac{1}{1 + (g_1 \vec{\pi'})^2}(1 + i g_1 \gamma_5 \vec{\tau}.\vec{\pi'}) N',$$

with $\vec{\pi'}$ a new pion field variable defined such that each pion-nucleon interaction involves at least one derivative term $\partial_\mu \vec{\pi'}$:

$$\vec{\pi'} = \frac{2\vec{\pi}}{1 - 2g_1\sigma + \left[(1 - 2g_1\sigma)^2 + 4g_1^2\vec{\pi}^2\right]^{1/2}}$$

If we take $m_\sigma \to \infty$ and introduce appropriate coefficients, we obtain (using π and N for simplicity):

$$\mathcal{L}_{\text{eff}} = -\bar{N}(\gamma^\mu \partial_\mu + m_N)N - \frac{1/2}{\left[1 + (\frac{g_V \vec{\pi}}{f_\pi})^2\right]^2} \partial_\mu \vec{\pi}.\partial^\mu \vec{\pi} - \frac{1/2}{1 + (\frac{g_V \vec{\pi}}{f_\pi})^2} m_\pi^2 \vec{\pi}^2$$

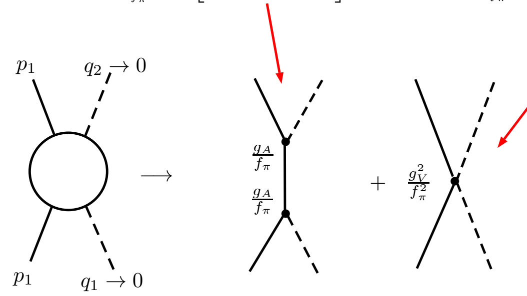
$$- \frac{1}{1 + (\frac{g_V \vec{\pi}}{f_\pi})^2} \bar{N} \left[\frac{g_A}{f_\pi} i\gamma^\mu \gamma_5 \vec{\tau}.\partial_\mu \vec{\pi} + \frac{g_V^2}{f_\pi^2} i\gamma^\mu \vec{\tau}.\left(\vec{\pi} \times \partial_\mu \vec{\pi}\right) \right] N$$

Phenomenological coefficients:

- For each soft pion: Multiply by f_π^{-1} (PCAC) and g_V (normalized current algebra commutators $[A_a^0, A_b^\mu] = \epsilon_{abc} V_c^\mu$);
- For each soft pion emitted through an axial vector current: Multiply by g_A/g_V (normalized vertex $\langle N' | A_a^\mu | N \rangle$).

In the simple $N+\pi \to N+\pi$ case:

$$\frac{1}{1+(\frac{g_V\vec{\pi}}{f_\pi})^2}\bar{N}\left[\frac{g_A}{f_\pi}i\gamma^\mu\gamma_5\vec{\tau}.\partial_\mu\vec{\pi}\right]N \qquad \frac{1}{1+(\frac{g_V\vec{\pi}}{f_\pi})^2}\bar{N}\left[\frac{g_V^2}{f_\pi^2}i\gamma^\mu\vec{\tau}.(\vec{\pi}\times\partial_\mu\vec{\pi})\right]N$$



At tree level $O(f_\pi^{-2})$
and for $O(q_i)$

In more complicated cases $(g_V f_\pi^{-1} \to f_\pi^{-1})$:

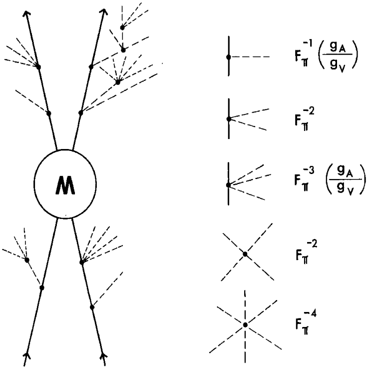
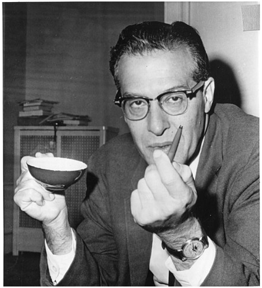


Fig. 2 Symbolic representation of current-algebra results for emission of 23 soft pions in nucleon-nucleon scattering. (Solid lines are nucleons, dashed lines pions.) The coupling constants associated with various vertices are shown on the right.


Schwinger (1965)

A few months after this work, Julian Schwinger remarked to me that it should be possible to skip this complicated derivation, forget all about the linear σ-model, and instead infer the structure of the Lagrangian directly from the non-linear chiral transformation properties of the pion field [...]. It was a good idea. I spent the summer of 1967 working out these transformation properties, and what they imply for the structure of the Lagrangian. (Weinberg, 2016, p. 5)

Nonlinear Realizations of Chiral Symmetry*

STEVEN WEINBERG†
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 25 September 1967)

We explore possible realizations of chiral symmetry, based on isotopic multiplets of fields whose transformation rules involve only isotopic-spin matrices and the pion field. The transformation rules are unique, up to possible redefinitions of the pion field. Chiral-invariant Lagrangians can be constructed by forming isotopic-spin-conserving functions of a covariant pion derivative, plus other fields and their covariant derivatives. The resulting models are essentially equivalent to those that have been derived by treating chirality as an ordinary linear symmetry broken by the vacuum, except that we do not have to commit ourselves as to the grouping of hadrons into chiral multiplets; as a result, the unrenormalized value of g_A/g_V need not be unity. We classify the possible choices of the chiral-symmetry-breaking term in the Lagrangian according to their chiral transformation properties, and give the values of the pion-pion scattering lengths for each choice. If the symmetry-breaking term has the simplest possible transformation properties, then the scattering lengths are those previously derived from current algebra. An alternative method of constructing chiral-invariant Lagrangians, using ρ mesons to form covariant derivatives, is also presented. In this formalism, ρ dominance is automatic, and the current-algebra result from the ρ-meson coupling constant arises from the independent assumption that ρ mesons couple universally to pions and other particles. Including ρ mesons in the Lagrangian has no effect on the π-π scattering lengths, because chiral invariance requires that we also include direct pion self-couplings which cancel the ρ-exchange diagrams for pion energies near threshold.

In a nutshell: Start with the most general chiral $SU(2)xSU(2)$ transformation rule for $\vec{\pi}(x)$ with generators T_a, X_a:

$$[T_a, \pi_b] = i\epsilon_{abc}\pi_c$$
$$[X_a, \pi_b] = -if_{ab}(\vec{\pi})$$

The axial vector transformation rule of $\vec{\pi}(x)$ is uniquely determined by its general properties (e.g., even parity) and the chiral commutators:

$$f_{ab}(\vec{\pi}) = \delta_{ab} f(\vec{\pi}^2) + \pi_a \pi_b \frac{1 + 2f(\vec{\pi}^2)f'(\vec{\pi}^2)}{f(\vec{\pi}^2) - 2\vec{\pi}^2 f'(\vec{\pi}^2)}$$

We can find a similar general transformation rule for any other field N ($[X_a, N] = v_{ab}(\vec{\pi})t_b N$) such that any isospin-invariant function of N is also chiral-invariant.

- Construct covariant derivatives $D_\mu \pi_a$ and $D_\mu N$ with the same axial transformation rule as N .

Upshot: Any arbitrary Lagrangian constructed out of $D_\mu \pi_a$, $D_\mu N$, and N will remain chiral-invariant if it is isospin-invariant.

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} D_\mu \vec{\pi}.D^\mu \vec{\pi} + \left[\frac{g_A}{f_\pi} \bar{N} i\gamma^\mu \gamma_5 \vec{\tau} N\right].D_\mu \vec{\pi} + \ldots$$

$$= \frac{1/2}{\left[1 + (\frac{g_V \vec{\pi}}{f_\pi})^2\right]^2} \partial_\mu \vec{\pi}.\partial^\mu \vec{\pi} + \frac{1}{1 + (\frac{g_V \vec{\pi}}{f_\pi})^2} \bar{N} \left[\frac{g_A}{f_\pi} i\gamma^\mu \gamma_5 \vec{\tau}.\partial_\mu \vec{\pi}\right] N + \ldots,$$

with a particular choice of pion field such that $D_\mu \vec{\pi} = \frac{1}{1+(g_V \vec{\pi}/f_\pi)^2} \partial_\mu \vec{\pi}$.

- Any such Lagrangian is equivalent for any on-shell amplitude (just a redefinition of the field variables).

Bottom-up EFT in 1966-1967?

1) Defining feature: $\mathcal{L}_{\text{eff}}$ embodies a set of symmetry principles (chiral invariance);

2) Form: Covariant derivative expansion in the pion field to systematically treat amplitudes with n soft pions on external lines at leading order;

3) Power-counting scheme: $1/f_\pi^N$ for N soft pions and external soft momenta contribution in $O(q^n)$;

4) Application: Restricted to tree-level amplitudes in the soft pion limit.

Why do we *really* need to introduce all the terms compatible with chiral invariance?

- We can always redefine the pion field $\pi^{'} = \pi + \delta\pi$ by means of a non-linear chiral transformation.

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In what sense does $\mathcal{L}_{\text{eff}}$ constitute a "phenomenological" Lagrangian?

- Parameters fixed by hand + tree-level amplitudes + low-energy limit.

Underlying justification?

- Easily reproduce the results of PCAC & current algebra for the emission and absorption of *N* soft pions.

3. Conclusion: Model independence in 1967?

Taking stock:

Weinberg's concept of phenomenological Lagrangian in 1966-1967 constitutes a distinctive prototype of bottom-up EFT (covariant derivative expansion).

However, no robust power-counting scheme to emancipate $\mathcal{L}_{\text{eff}}$ from PCAC & current algebra and go beyond tree level (i.e., $\mathcal{L}_{\text{eff}}$ not used as a dynamical model).

Why introducing $\mathcal{L}_{\text{eff}}$ in the first place?

Pragmatic answer: To derive more easily and systematically the results of PCAC & current algebra.

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Yet two decisive historical elements:

- From 1954 onwards: We need to rely on basic principles (e.g., symmetries) to select relevant interaction terms and reduce the set of potentially relevant models.
- From 1963 onwards: We need to include all the possible interaction terms compatible with these principles if we want to account systematically for arbitrary processes.

Postcript:

1967-1971: Systematic development and extension of phenomenological Lagrangians + first attempts to evaluate loop contributions (e.g., Weinberg, 1970; Gerstein et al., 1971).

1971-1974: The "renaissance" of field theory somewhat put this endeavor to a halt;

Mid-1970s: Non-renormalizable theories can be systematically renormalized + renormalizability is not a pristine principle;

1979: First systematic power-counting scheme for chiral perturbation theory (Weinberg, 1979).

Model independence in 1968?

- The choice of a particular Lagrangian does not matter; any model satisfying the relevant set of principles will do (e.g., chirality and PCAC & current algebra compatible).

- The particular definition of fields does not really matter too; they only need to have appropriate chiral transformation properties.

- $\mathcal{L}_{\text{eff}}$ is restricted to low energies; we don't need to know what is the correct high-energy model.

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- The particular definition of fields does not really matter too; they only need to have appropriate chiral transformation properties.
- $\mathcal{L}_{\text{eff}}$ is restricted to low energies; we don't need to know what is the correct high-energy model.

Key missing element: Without a robust power-counting rule, $\mathcal{L}_{\text{eff}}$ is not yet tailored to systematically account for high-energy effects at low energies.

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