

Atmospheric flux calculations and LHC input

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based on work in collaboration with

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How to get atmospheric fluxes? From cascade equations to Z -moments [review in Gaisser, 1990; Lipari, 1993]

Solve a system of **coupled differential equations** regulating particle evolution in the atmosphere (interaction/decay/(re)generation):

$$\frac{d\phi_j(E_j, X)}{dX} = -\frac{\phi_j(E_j, X)}{\lambda_{j,int}(E_j)} - \frac{\phi_j(E_j, X)}{\lambda_{j,dec}(E_j)} + \sum_{k \neq j} S_{prod}^{k \rightarrow j}(E_j, X) + \sum_{k \neq j} S_{decay}^{k \rightarrow j}(E_j, X) + S_{reg}^{j \rightarrow j}(E_j, X)$$

Under assumption that X dependence of fluxes factorizes from E dependence, analytical approximated solutions in terms of Z -moments:

– **Particle Production:**

$$S_{prod}^{k \rightarrow j}(E_j, X) = \int_{E_j}^{\infty} dE_k \frac{\phi_k(E_k, X)}{\lambda_k(E_k)} \frac{1}{\sigma_k} \frac{d\sigma_{k \rightarrow j}(E_k, E_j)}{dE_j} \sim \frac{\phi_k(E_j, X)}{\lambda_k(E_j)} Z_{kj}(E_j)$$

– **Particle Decay:**

$$S_{decay}^{j \rightarrow l}(E_l, X) = \int_{E_l}^{\infty} dE_j \frac{\phi_j(E_j, X)}{\lambda_j(E_j)} \frac{1}{\Gamma_j} \frac{d\Gamma_{j \rightarrow l}(E_j, E_l)}{dE_l} \sim \frac{\phi_j(E_l, X)}{\lambda_j(E_l)} Z_{jl}(E_l)$$

Solutions for $E_j \gg E_{crit,j}$ and for $E_j \ll E_{crit,j}$, respectively, are interpolated geometrically.

Atmospheric neutrino fluxes

CR + Air interactions:

- *AA'* interaction approximated as *A NA'* interactions (super position);
- *NA'* approximated as *A' NN* interactions: up to which extent is this valid ?

* conventional neutrino flux:

$$NN \rightarrow u, d, s, \bar{u}, \bar{d}, \bar{s} + X \rightarrow \pi^\pm, K^\pm + X' \rightarrow \nu_\ell(\bar{\nu}_\ell) + \ell^\pm + X',$$

$$NN \rightarrow u, d, s, \bar{u}, \bar{d}, \bar{s} + X \rightarrow K_S^0, K_L^0 + X \rightarrow \pi^\pm + \ell^\mp + \nu_{(-)} + X$$

$$NN \rightarrow u, d, s, \bar{u}, \bar{d}, \bar{s} + X \rightarrow \text{light hadron} + X' \rightarrow \nu(\bar{\nu}) + X''$$

* prompt neutrino flux:

$$NN \rightarrow c, b, \bar{c}, \bar{b} + X \rightarrow \text{heavy-hadron} + X' \rightarrow \nu(\bar{\nu}) + X'' + X'$$

where the decay to neutrino occurs through semileptonic and leptonic decays:

$$D^+ \rightarrow e^+ \nu_e X, \quad D^+ \rightarrow \mu^+ \nu_\mu X,$$

$$D_s^\pm \rightarrow \nu_\tau(\bar{\nu}_\tau) + \tau^\pm, \quad \text{with further decay } \tau^\pm \rightarrow \nu_\tau(\bar{\nu}_\tau) + X$$

proper decay lengths: $c\tau_{0, \pi^\pm} = 780 \text{ cm}$, $c\tau_{0, K^\pm} = 371 \text{ cm}$, $c\tau_{0, D^\pm} = 0.031 \text{ cm}$

Critical energy $\epsilon_h = m_h c^2 h_0 / (c \tau_{0, h} \cos(\theta))$, above which hadron **decay** probability is suppressed with respect to its **interaction** probability:

$\epsilon_\pi^\pm < \epsilon_K^\pm \ll \epsilon_D \Rightarrow$ conventional flux is suppressed with respect to prompt one, for energies high enough, due to finite atmosphere height h_0 .

Flavour Number Schemes for heavy-flavour production

$m_l = m_u, m_d, m_s < \Lambda_{QCD} \rightarrow$ assumed massless always,

$m_{HQ} = m_c, m_b, m_t > \Lambda_{QCD}$ treatment depends on scheme

and, depending on the kinematics and \sqrt{s} , it may happen $p_{T,HQ} \gg m_Q$ or $Q \gg m_Q$

* **Decoupling scheme with a fixed flavour number (FFNS):**

- The mass of at least one (or more) HQ is retained at all scales
- HQ can be produced as final states and circulate in loops.
- They are excluded from initial states.
- Divergences due to light quark loops contributing to α_S renormalization are subtracted at zero mass (like in the \overline{MS} scheme), those due to heavy-quark loops are subtracted at zero momentum.
- **issue** at high p_T or Q : $\log(p_{T,HQ}^2/m_{HQ}^2)$ or $\log(Q^2/m_{HQ}^2)$ may become so big that they may spoil the convergence of the perturbative series!

* **Zero-mass variable flavour number scheme (ZM-VFNS):**

- HQ massless quarks at all scales in all components of the calculation.
- These quarks are present in the initial state above fixed thresholds.
- They contribute to α_S running (in the \overline{MS} scheme) above the same thresholds.
- **issue** at low p_T or Q : powers of $(m_{HQ}^2/p_{T,HQ}^2)$ or of (m_{HQ}^2/Q_{HQ}^2) missing!

* **General-mass variable flavour number schemes (GM-VFNS):**

- HQ mass retained in part of the calculation;
meant to combine optimal features of FFNS and ZM-VFNS at different p_T or Q .
- **advantage**: logs resummed and powers (at least the leading ones) present.
- **problem**: some arbitrariness in the combination (different variants possible)

QCD factorization

for 1-particle inclusive heavy-hadron hadroproduction

$$d\sigma(N_1 N_2 \rightarrow H + X) = \sum_{abc} PDF_a^{N_1}(x_a, \mu_{F,i}) PDF_b^{N_2}(x_b, \mu_{F,i}) \otimes \\ \otimes d\hat{\sigma}_{ab \rightarrow cX'}(x_a, x_b, z, \mu_{F,i}, \mu_{F,f}, \mu_R) \otimes FF_c^H(z, \mu_{F,f})$$

$d\hat{\sigma}$: differential perturbative partonic cross-section,
its m_h dependence, neglected in ZM-VFNS, is instead kept in GM-VFNS.

μ_F, μ_R reabsorb IR and UV divergences (truncation of P.T. series).

PDFs: perturbative evolution with factorization scale $\mu_{F,i}$,
non-perturbative dependence on $x = p^+ / P_N^+$.

FFs: perturbative evolution with factorization scale $\mu_{F,f}$,
non perturbative parameterization in terms of $z = P_H^+ / p_c^+$ frequently used.

QCD uncertainties

- * $\mu_{F,i}, \mu_{F,f}$ and μ_R choice: no univocal recipe.
- * Approximate knowledge of heavy-quark mass values m_h (SM input parameters).
- * Choice of Variable Flavour Number Scheme (several possibilities!)
- * PDF (+ $\alpha_S(M_Z)$) fits to experimental data
- * Fits to experimental data of the non-perturbative parameters of the FF

GM-VFNS partonic and hadronic pp cross-sections

* Taking the limit $m \rightarrow 0$ of the massive cross-section does not reduce to the massless one obtained in dimensional regularization for $\epsilon \rightarrow 0$ (finite mass regulators and dimensional regularization of collinear divergences yield different finite terms):

$$d\hat{\sigma}^{sub} = \lim_{m \rightarrow 0} d\hat{\sigma}^{FFNS}(m) - d\hat{\sigma}^{ZM-VFNS}$$

* Subtraction of overlapping terms:

$$d\hat{\sigma}^{GM-VFNS}(m) = d\hat{\sigma}^{FFNS}(m) - a d\hat{\sigma}^{sub}$$

with $a = 1$ in the S-ACOT scheme,

$a = a(p_T, m, \dots)$ for other schemes (e.g. FONLL).

* S-ACOT GM-VFNS hadronic σ for single inclusive heavy-hadron production obeys a factorization formula which generalizes the CSS factorization theorem in the ZM-VFNS:

$$d\sigma_{pp \rightarrow hX}^{ZM}(P, S) = F_{i/p}(x_1, \mu_i) F_{j/p}(x_2, \mu_j) \otimes d\hat{\sigma}_{ij \rightarrow kX}(p, s, \mu_r, \mu_i, \mu_j) \otimes D_{h/k}(z, \mu_f)$$

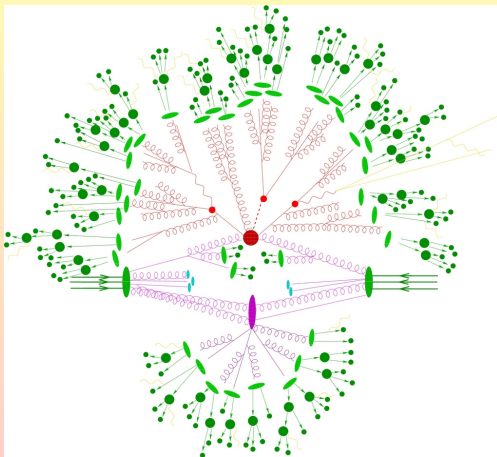
$$\frac{1}{p_T} \frac{d\sigma_{pp \rightarrow hX}^{ZM}}{dp_T dy}(S, p_T, y) = \frac{2}{S} \sum_{i,j,k} \int_{1-v+vw}^1 \frac{dz}{z^2} \int_{\frac{vw}{z}}^{1-\frac{1-v}{z}} \frac{dv}{1-v} \int_{\frac{vw}{vz}}^1 \frac{dw}{w}$$

$$F_{i/p}(x_1, \mu_i) F_{j/p}(x_2, \mu_j) \frac{1}{v} \frac{d\hat{\sigma}_{ij \rightarrow kX}^{ZM}}{dv dw}(s, v, w, z, \mu_r, \mu_i, \mu_j) D_{h/k}(z, \mu_f),$$

with proper replacement of massless kinematics and integration limits with massive kinematics and integration limits where needed.

Alternative: NLO matched to PS in SMC

p-p and p-p̄ collision overview (LHC and Tevatron)

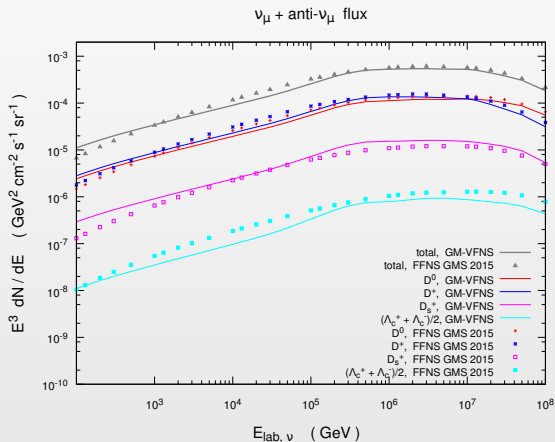


- $Q = a \text{ few TeV}$
- hard scattering
 - parton shower
 - QED shower
 - hadronization $\Lambda_{QCD} = 200 \text{ MeV}$
 - hadron decay
 - underlying event
 - pile-up (overlap of different collisions).

DECREASING ENERGY SCALE

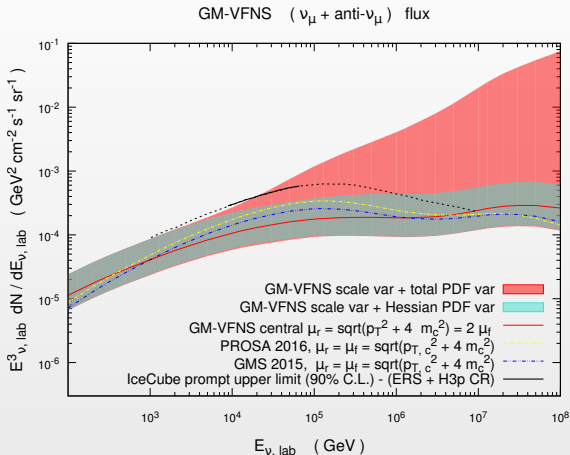
PERTURBATIVE AND NON-PERTURBATIVE COMPONENTS

Prompt ($\nu_\mu + \bar{\nu}_\mu$) fluxes in the GM-VFNS and in GMS (i.e. NLO + PYTHIA) approach: contribution of different charmed hadrons



Prompt neutrino fluxes with GM-VFNS:

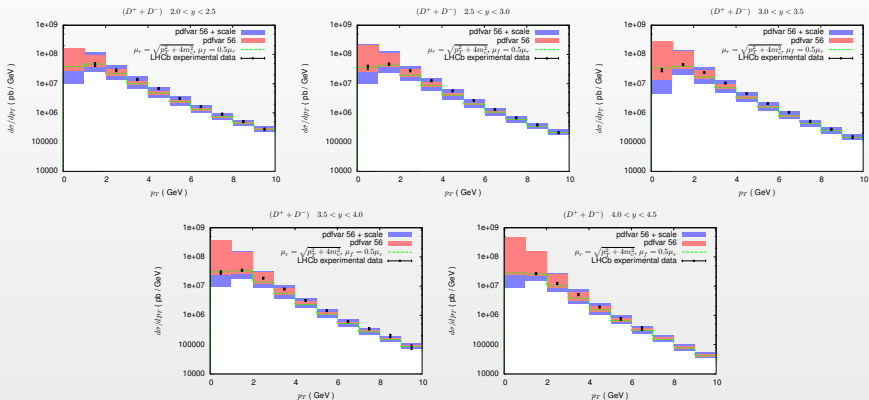
theoretical predictions from [arXiv:1705.10386] vs. IceCube upper limits



The extrapolation to high energy of IceCube results suggest that the CT14nlo gluon PDF uncertainty band at low x 's is too large!

GM-VFNS predictions vs experimental data

for differential cross-sections for $pp \rightarrow D^\pm + X$ at LHCb at 5 TeV



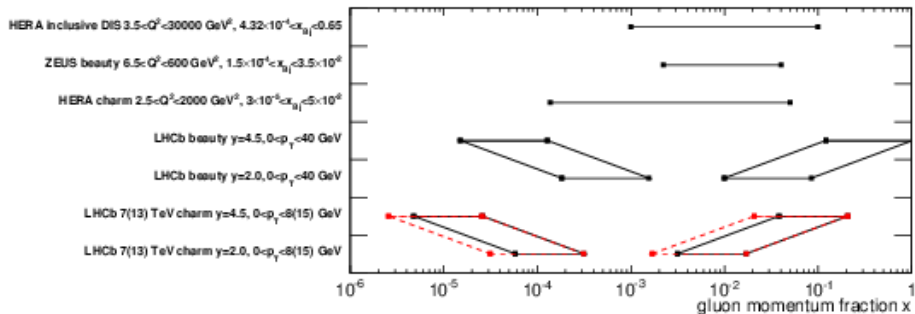
* Scale uncertainties larger than experimental ones.

* Large PDF uncertainties (CT14nlo), increasing at low p_T / large y , can be mitigated by using other PDF fits.

Important ingredient of these calculations: PDF fits

- * x -dependence of PDFs is fitted to experimental data.
- * HERA data (core of all PDF fits) offer good coverage in the range $10^{-4} < x < 10^{-1}$.
- * Prompt ν fluxes sensitive to a wider range of x values, due to the fact that
 - The \sqrt{s} for the relevant collisions involve a wide range of energies (from $\sqrt{s} \sim 100$ GeV to $\sqrt{s} \sim 150$ TeV).
 - The relevant rapidities extend to values much larger than those accessible in traditional experiments at human-made colliders.

x coverage of HERA and LHCb experiments

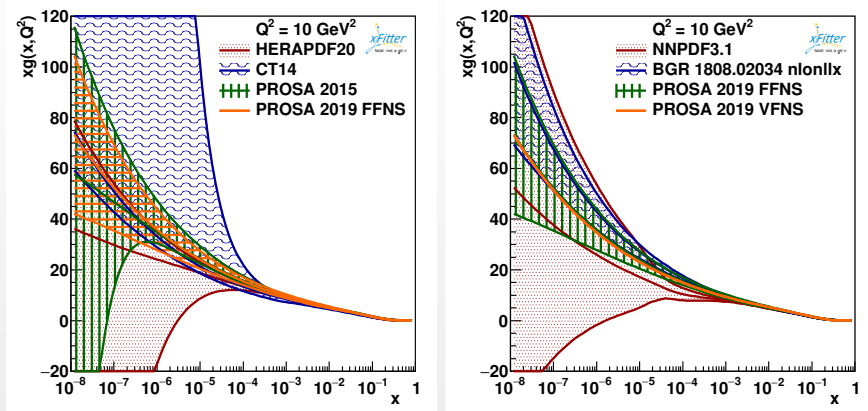


LHCb data allow to cover x regions uncovered by HERA data, both at low x 's (especially open charm data) and at large x 's (especially open bottom data).

For LHCb, LO formula $x_{1,2} = (\sqrt{p_T^2 + m_Q^2}/E_p) \exp(\pm y) \Rightarrow$ Larger rapidities of the emitted quark correspond to more extreme x 's; large $\sqrt{s} \leftrightarrow$ small x 's

charm production in DIS at EIC extends HERA charm coverage even to $x > 0.1$.

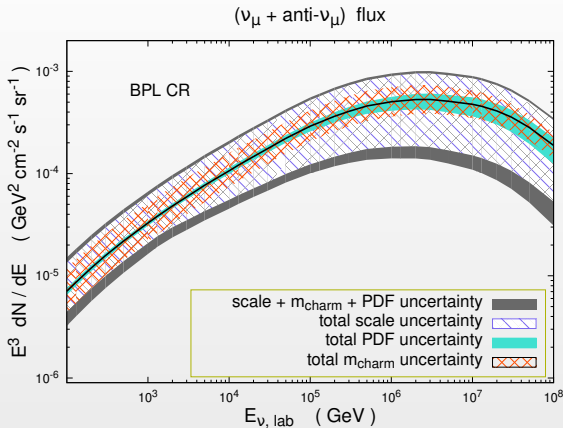
gluon PDF: comparison between different PDF fits



* PROSA2019 better constrained than PROSA2015 due to inclusion of data at $\sqrt{s} = 13$ and 5 TeV , besides 7 TeV .

* Compatibility between different PDF sets including D -meson LHCb data.

PROSA 2019 atmospheric prompt ($\nu_\mu + \bar{\nu}_\mu$) flux: QCD scale, mass and PDF uncertainties

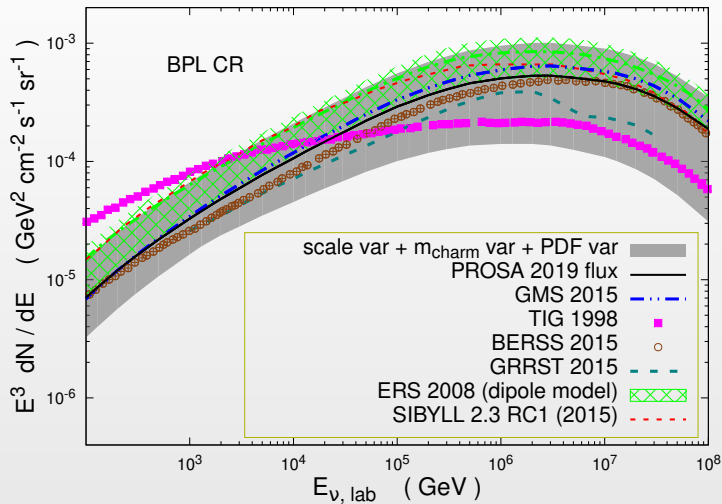


from [arXiv:1911.13164]

* PDF uncertainty subdominant,
assuming extrapolation at $x < 10^{-6}$ works.

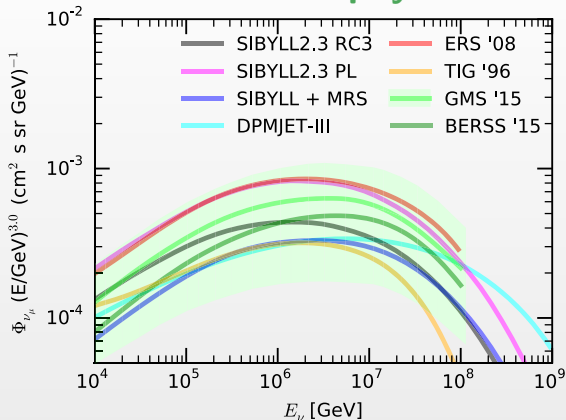
Comparison of predictions by different groups

$(\nu_\mu + \text{anti-}\nu_\mu)$ flux



Different predictions compatible within the uncertainty band

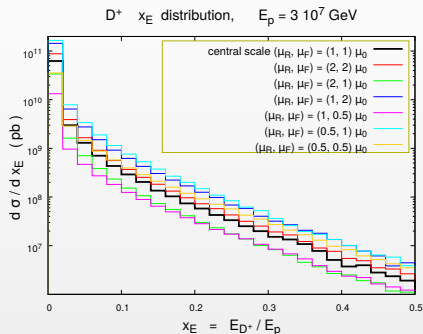
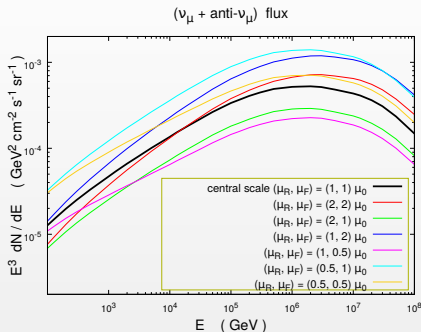
$(\nu_\mu + \bar{\nu}_\mu)$ fluxes: comparison with predictions of hadronic models used in EAS physics



from A. Fedynitch, EPJ Web of Conferences 116, 11010 (2016)

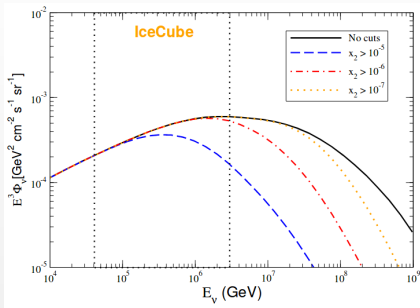
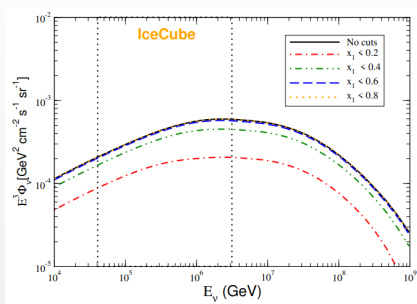
All **recent central predictions**, both those on the basis of pQCD and those on the basis of hadronic models used in EAS physics (like SIBYLL, DPMJET), **turn out to lie within our uncertainty band**.

μ_R and μ_F scale uncertainties



- * Scale uncertainties are evaluated by making an envelope over different variations.
- * Predictions have a shape uncertainty, not only a normalization uncertainty!

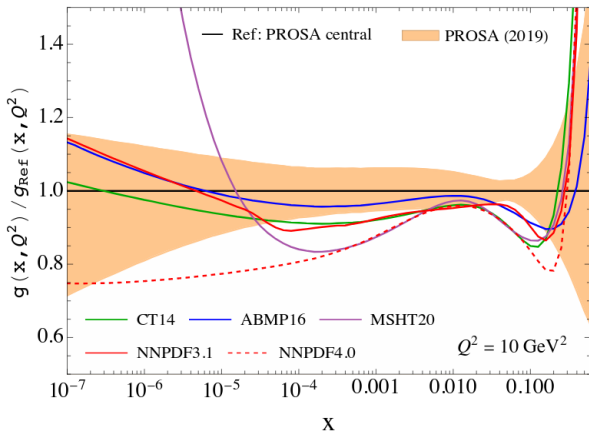
Prompt atmospheric ν fluxes, small- x and large- x PDFs



from V. Goncalves et al. [[arXiv:1708.03775](https://arxiv.org/abs/1708.03775)]

- * A robust estimate of large x effects is important for determining the normalization of prompt atmospheric neutrino fluxes
- * Region particularly relevant: $0.2 < x < 0.6$, partly testable through ν experiments at the LHC.
- * On the other hand, for ν at the PeV scale, knowledge of PDF down to $x > 10^{-6}$ is enough.

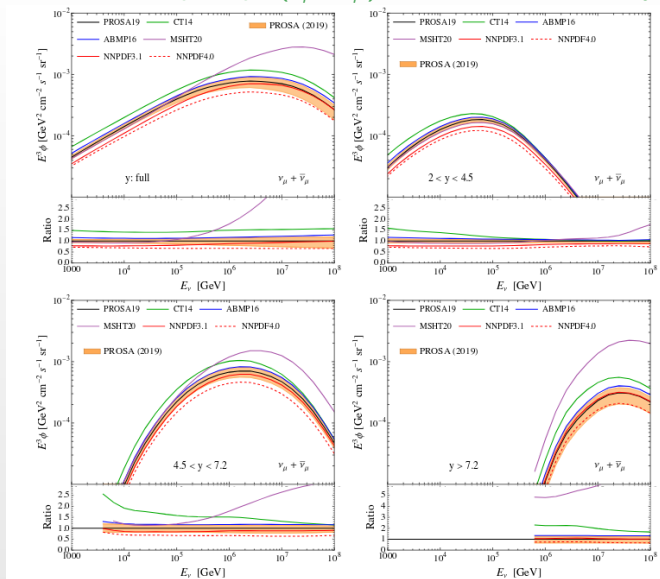
PDFs uncertainties at low and large- x



W. Bai et al., [arXiv:2212.07865]

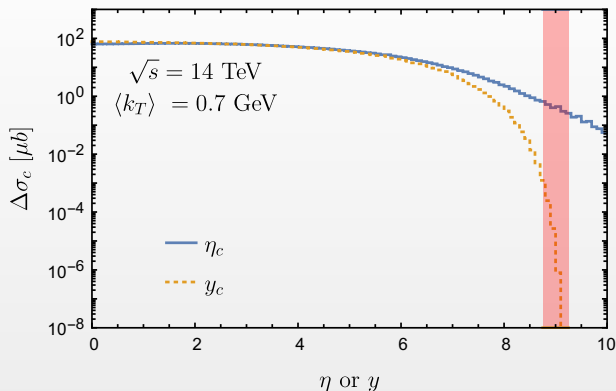
- * Differences in gluon PDFs at large x are not covered by the uncertainties associated to each single PDF set.

Contribution to prompt ($\nu_\mu + \bar{\nu}_\mu$) fluxes from different y_c regions



larger E_ν are obtained for larger \sqrt{s} , larger y and larger x .

Charm production at large rapidity/pseudorapidity



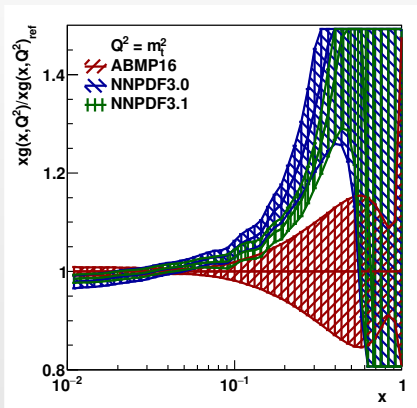
- * For forward charm production ($\eta_c \gtrsim 6$) rapidity and pseudorapidity distributions increasingly differ.
- * η_c distribution effectively limited by the fact that y_c distribution is bounded.

PDF uncertainties at large x

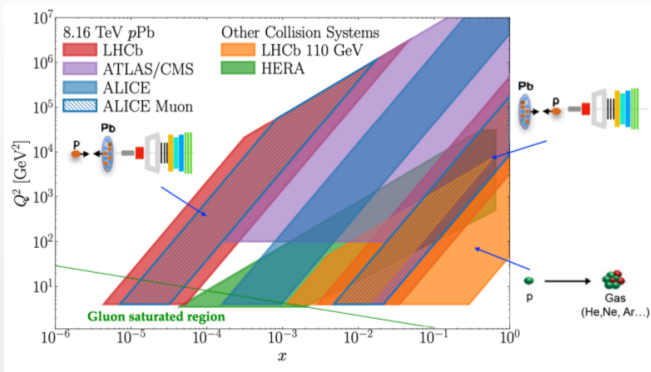
- * PDF uncertainties are often estimated by considering a single PDF set.
 - * However, the differences between different PDF sets might be not covered by the uncertainty of a single set.
- ⇒ A more comprehensive estimate would be recommended.

* g PDF at large x play an important role in the predictions.

⇒ data on $t\bar{t} + X$ and jet production at the LHC are important for constraining g PDF in this region.



Fixed-target experiments at the LHC: increased large x coverage and sensitivity to nuclear matter effects



from LHCb collaboration

* LHCb-FT coverage at scale $Q^2 \sim 4 \text{ GeV}^2$:

$$2 \cdot 10^{-4} \lesssim x \lesssim 4 \cdot 10^{-1} \Rightarrow \text{gluon, sea quarks}$$

* Light targets: probe NM effects in pA collisions in A range different from Pb

* Cold and Hot Nuclear Matter effects (at small x) can be compared by using p or Pb beams impinging on the nuclear targets (He, Ne, Ar,).

Nuclear modification factors R_p^{Pb}

* **Shadowing**: $R < 1$ for $x \lesssim 0.1$ (a possible explanation: parton recombination/fusion process enhanced in nuclear target: partons with large spatial uncertainties (small x), can leak to a neighbor nucleon)

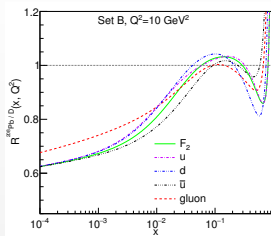
* **Antishadowing**: $R > 1$ for $0.1 \lesssim x \lesssim 0.3$, related to shadowing.

* **EMC effect**: $R < 1$ for $0.3 \lesssim x \lesssim 0.7$ (attributed to in-medium nucleon swelling, nucleon-nucleon short range correlations, binding, ...).

* **Fermi smearing**: $R > 1$ for $0.7 \lesssim x < A$ short range nucleon correlations deform the nuclear structure functions mainly at large x .

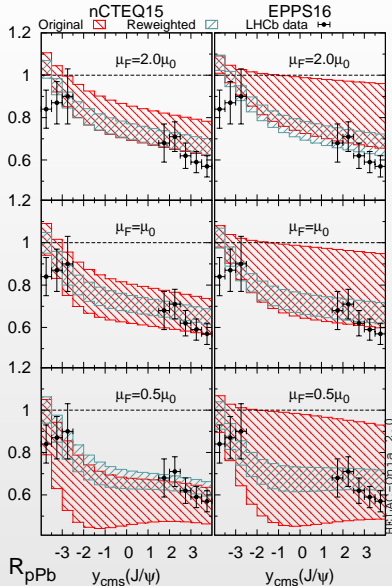
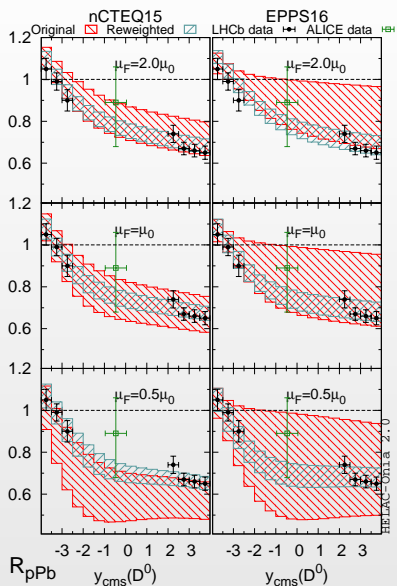
No explicit modelization of nuclear effects occurs in most global fits of nPDFs. The modifications of the structure functions by nuclear effects are **absorbed into the nPDF themselves**.

⇒ Evergreen questions: how to write a parameterization for nPDFs ?



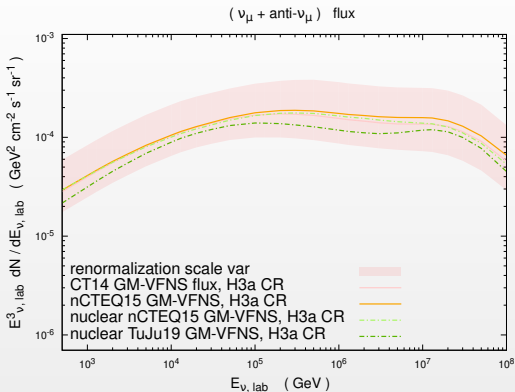
from [arXiv:1611.03670]

nPDF fits and D^0 and J/ψ data in p -Pb / pp collisions



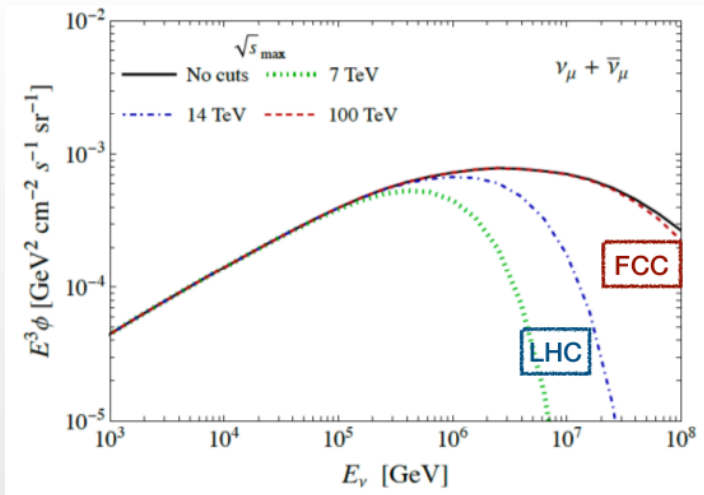
from A. Kusina, J.-P. Lansberg, I. Schienbein, H.-S. Shao, [arXiv:1712.07024v2]

$(\nu_\mu + \bar{\nu}_\mu)$ fluxes: cold nuclear matter effects



- * Predictions using nuclear PDFs within scale uncertainty bands of those with proton PDFs and superposition model.
- * Suppression of prompt fluxes due to CNM effects ?
Large shadowing effects do not emerge for all nuclear PDF fits, especially for low-mass nuclei

Prompt atmospheric ν fluxes and LHC phase-space coverage



* The $\sqrt{s} = 14$ TeV at LHC is in any case a limitation, FCC would be better (see also analysis in V. Goncalves et al, [arXiv:1708.03775]).

Wishlist useful measurements LHC, especially LHCb

- * D -meson and B -meson spectra at 13.6 TeV, 14 TeV.
- * if possible, more p_T bins in the region 0 - 5 GeV
- * Λ_c^\pm double-differential spectra in y, p_T .
- * Additional focus on D_s^\pm (main source of ν_τ and $\bar{\nu}_\tau$).
- * Charge asymmetries with better statistics.
- * All above in pp, pPb, pO standard collider modality
+ SMOG fixed-target modality using various light targets.
- * LHCb measurements of DY and $t\bar{t}$ -pair production in pp .
- * Measurements should be accompanied by detailed information concerning systematic uncertainties (correlation matrices).
- * Further measurements of correlations between D -mesons from c and \bar{c} help to stress-test theory predictions and to test predictions in factorization schemes beyond collinear one.