The MCEq code and atmospheric leptons

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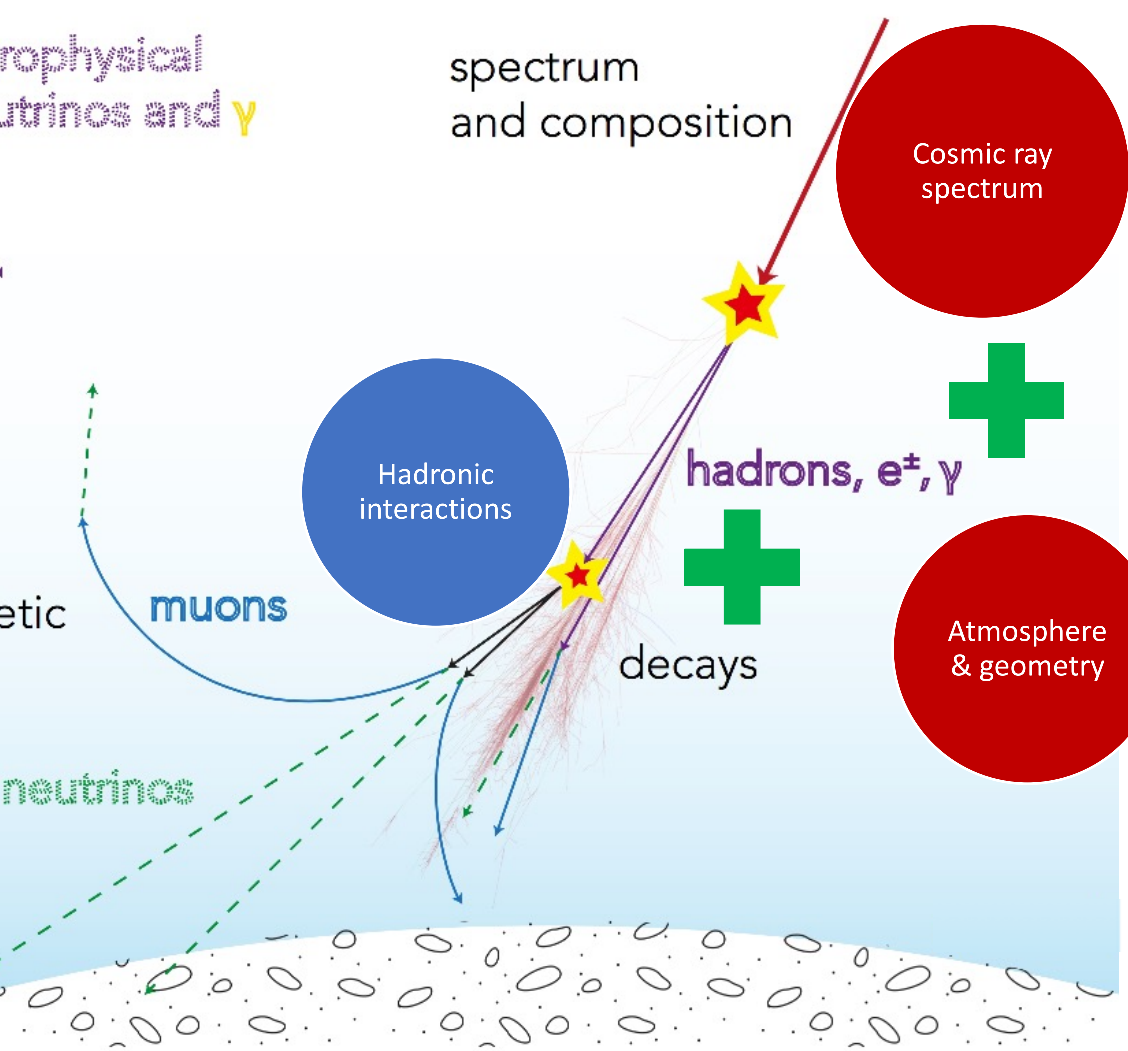
Workshop on the tuning of hadronic interaction models, Wuppertal, 2024/01/23

Modeling inclusive leptons in the atmosphere

“inclusive” = integrated over CR energy and all other particles at the surface



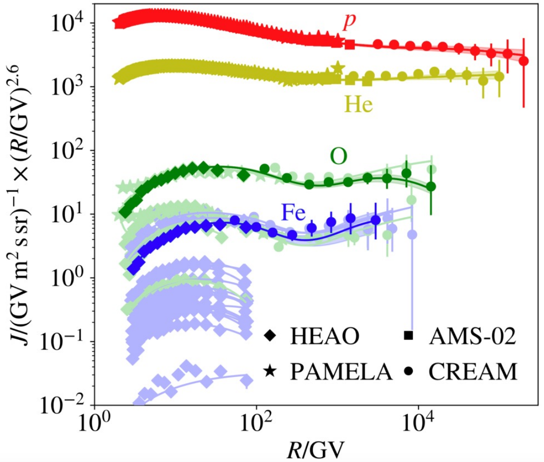
Cosmic ray spectrum

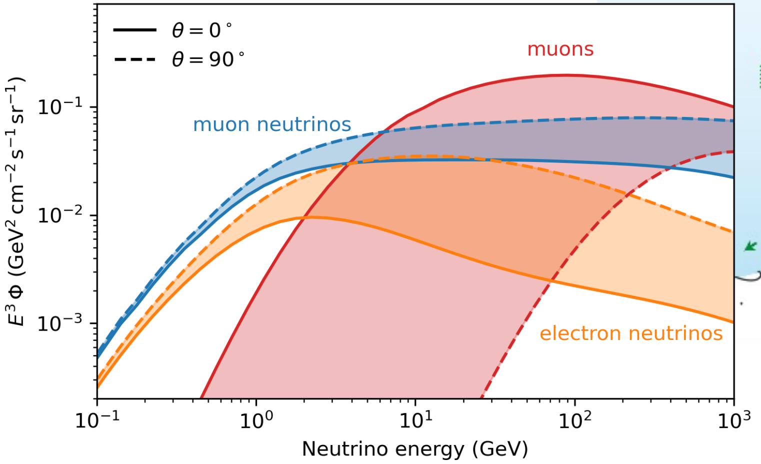
+

Hadronic interactions

+

Atmosphere & geometry



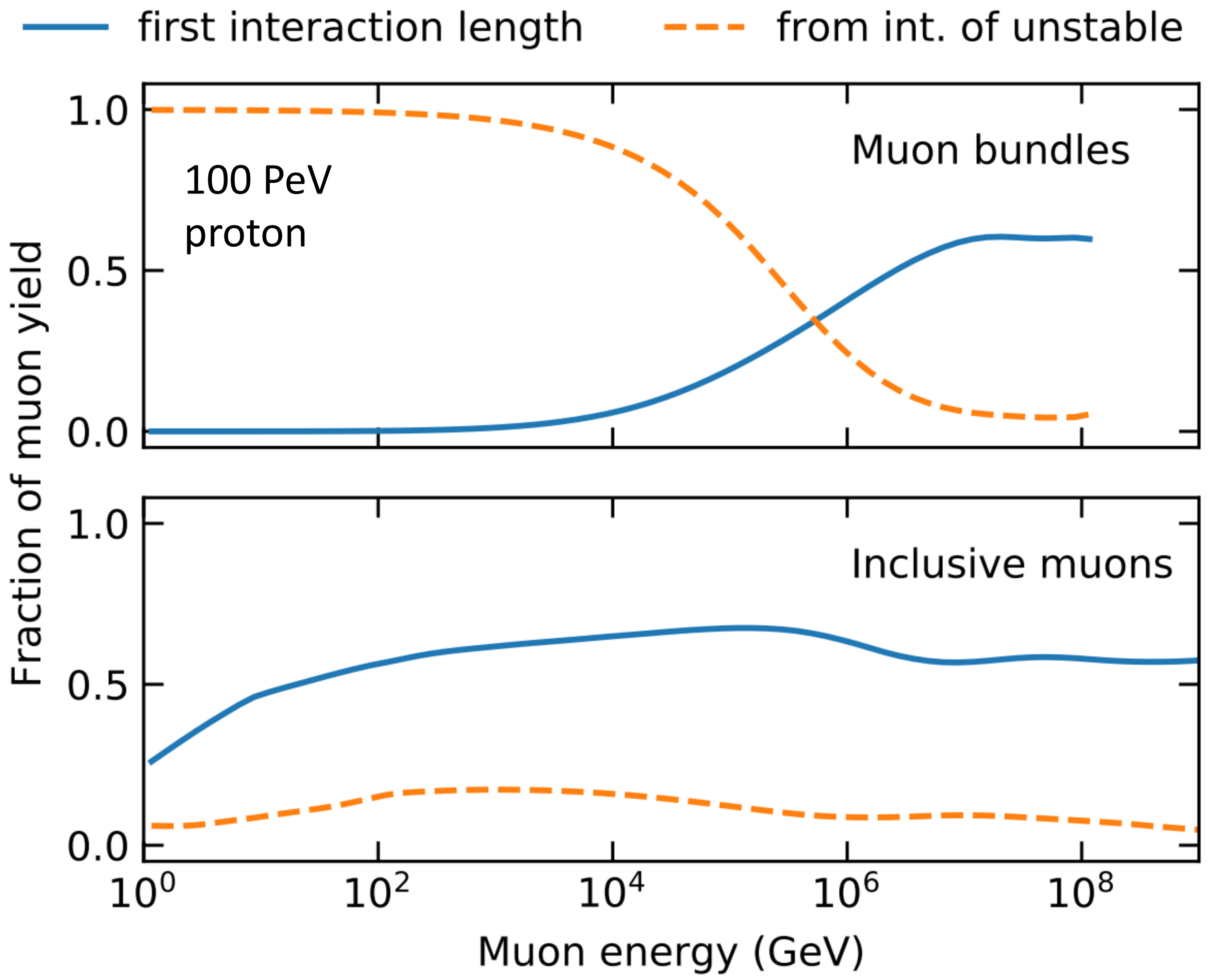


None of these models is founded on a “fundamental” theory/framework.

All are “theory-motivated”, “data-driven”, or empirical.

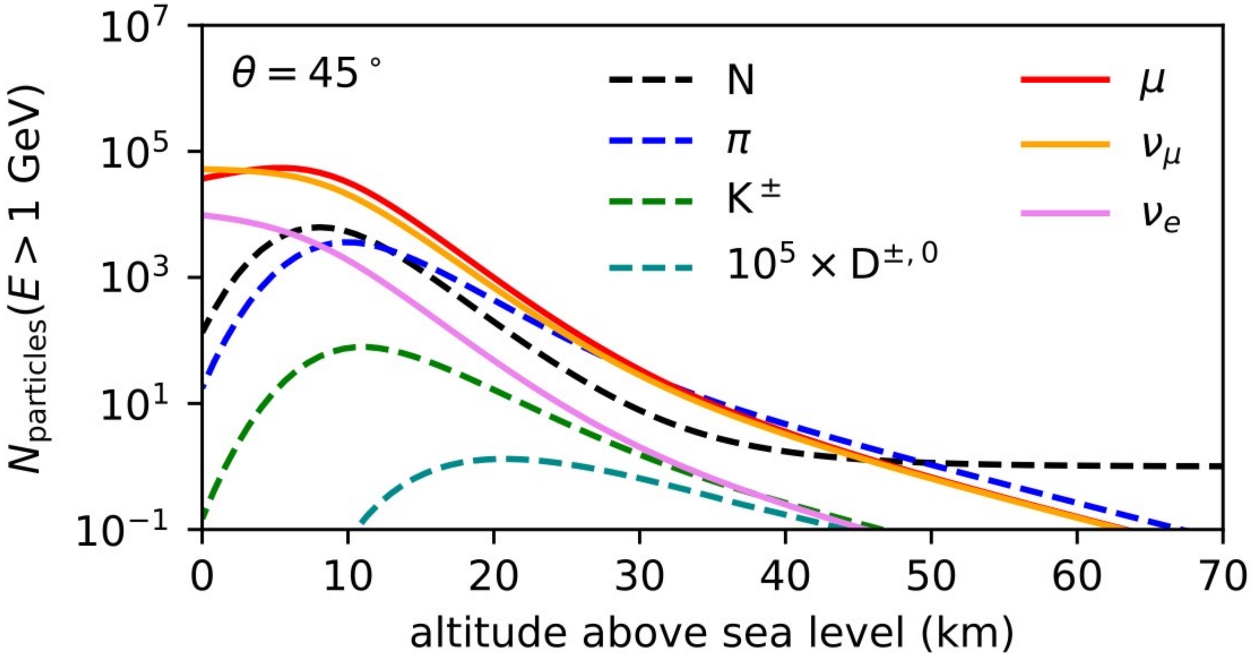
Atm. leptons != air showers: different “astroparticle observable”

- Inclusive fluxes sensitive to ”first interaction”
- Air shower muons at the surface mostly from pion interactions
- Reason: competition between falling CR flux vs falling forward cross section
- Problems in incl. leptons distinct should be distinct from air showers



What is MCEq?

1. Open-source iterative cascade equation solver
2. Cascade equations = transport equations (solved by CORSIKA using a Monte Carlo method)
3. Mainly used in atmospheric lepton and neutrino telescope community
4. Potentially interesting for
    - Atmospheric leptons > 1 GeV
    - Underground muons
    - Cascade eqn. solver in CORSIKA8
    - Air shower & cosmic ray "theory"
    - Beyond standard model/Pheno
    - Astrophysics



Transport equations (hadronic cascade equations) in 1D (and 2D)

System of coupled non-linear PDE for each particle species *h* :

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} - \frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} - \frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{int},k}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k \frac{\mathrm{d}N^{\mathrm{dec}}_{k(E_k)\to h(E)}}{\mathrm{d}E}\frac{\Phi_k(E_k,X)}{\lambda_{\mathrm{dec},k}(E_k,X}$$

cosmic ray physics

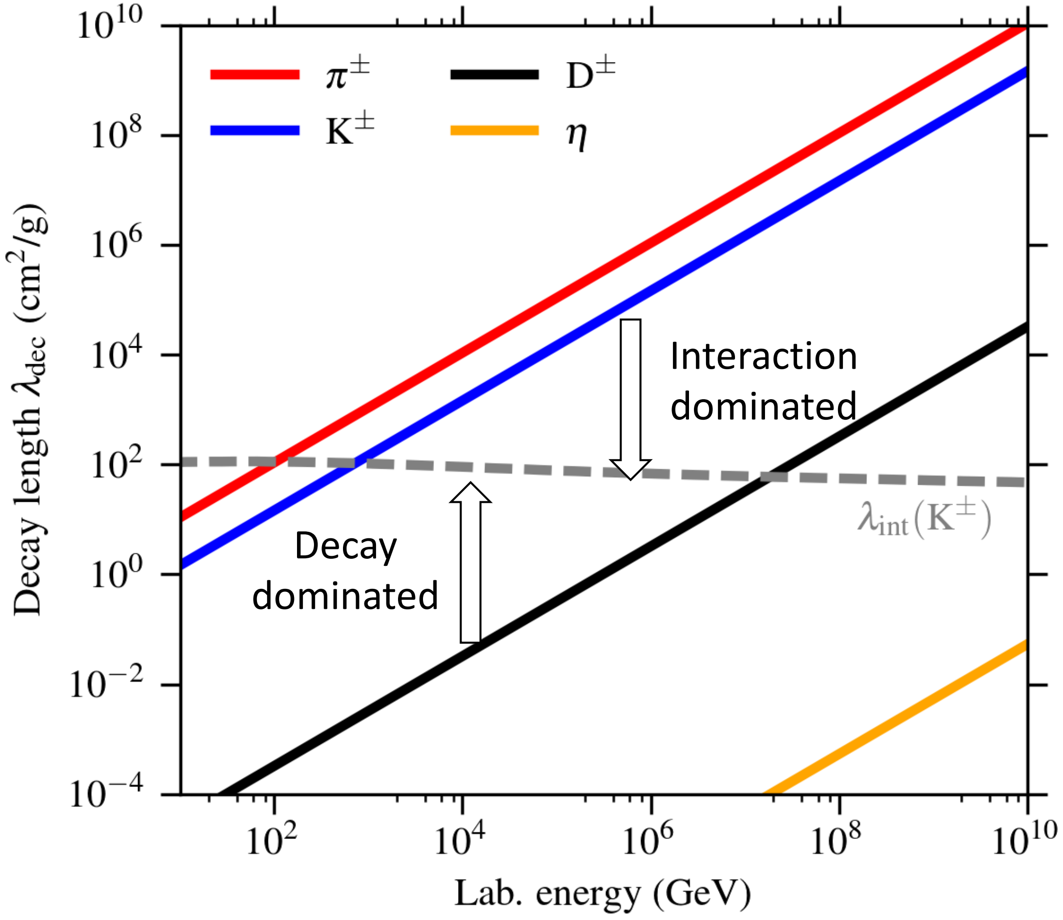
Interactions with air

Decays

atmospheric physics

Continuous losses

particle physics

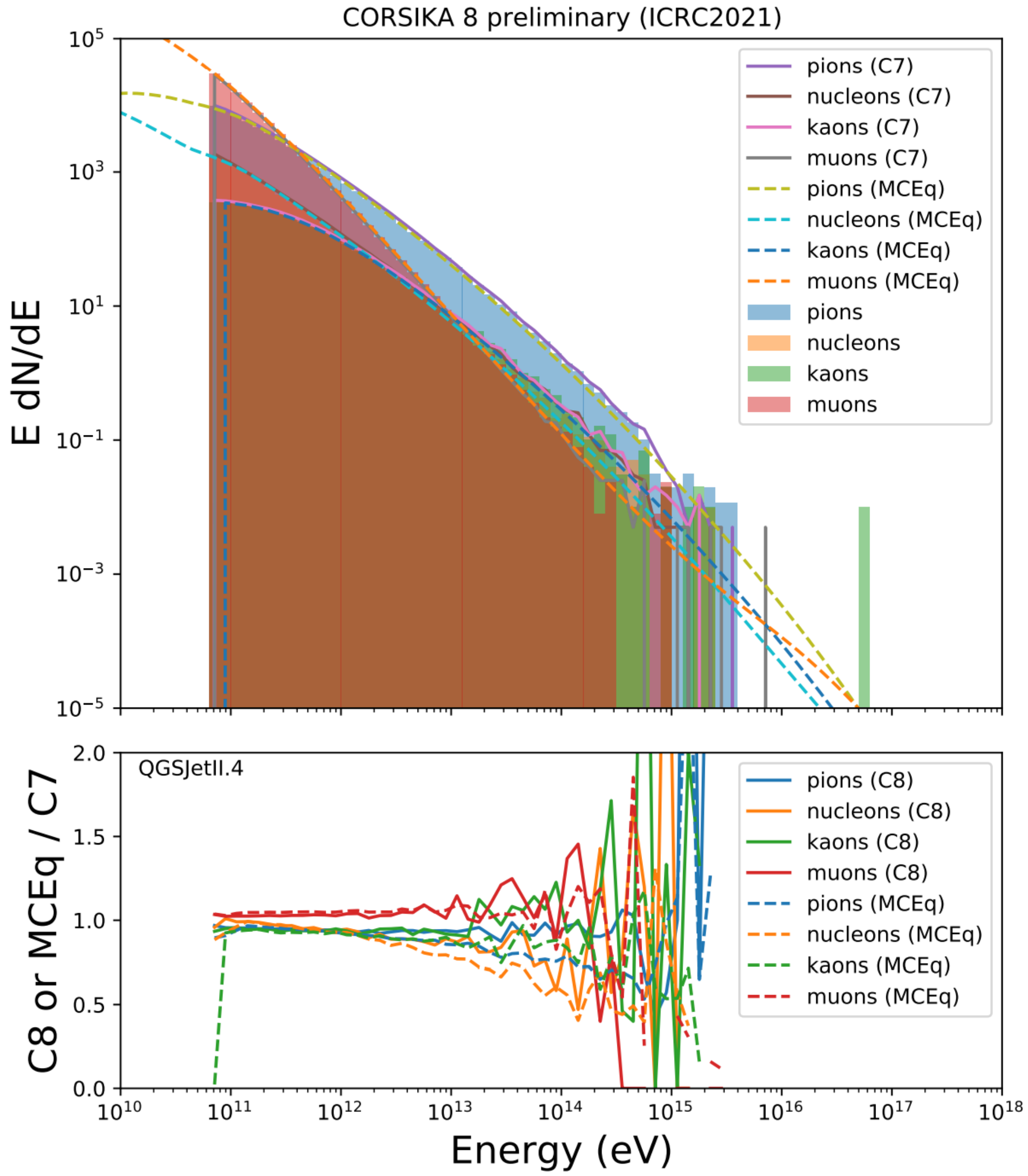


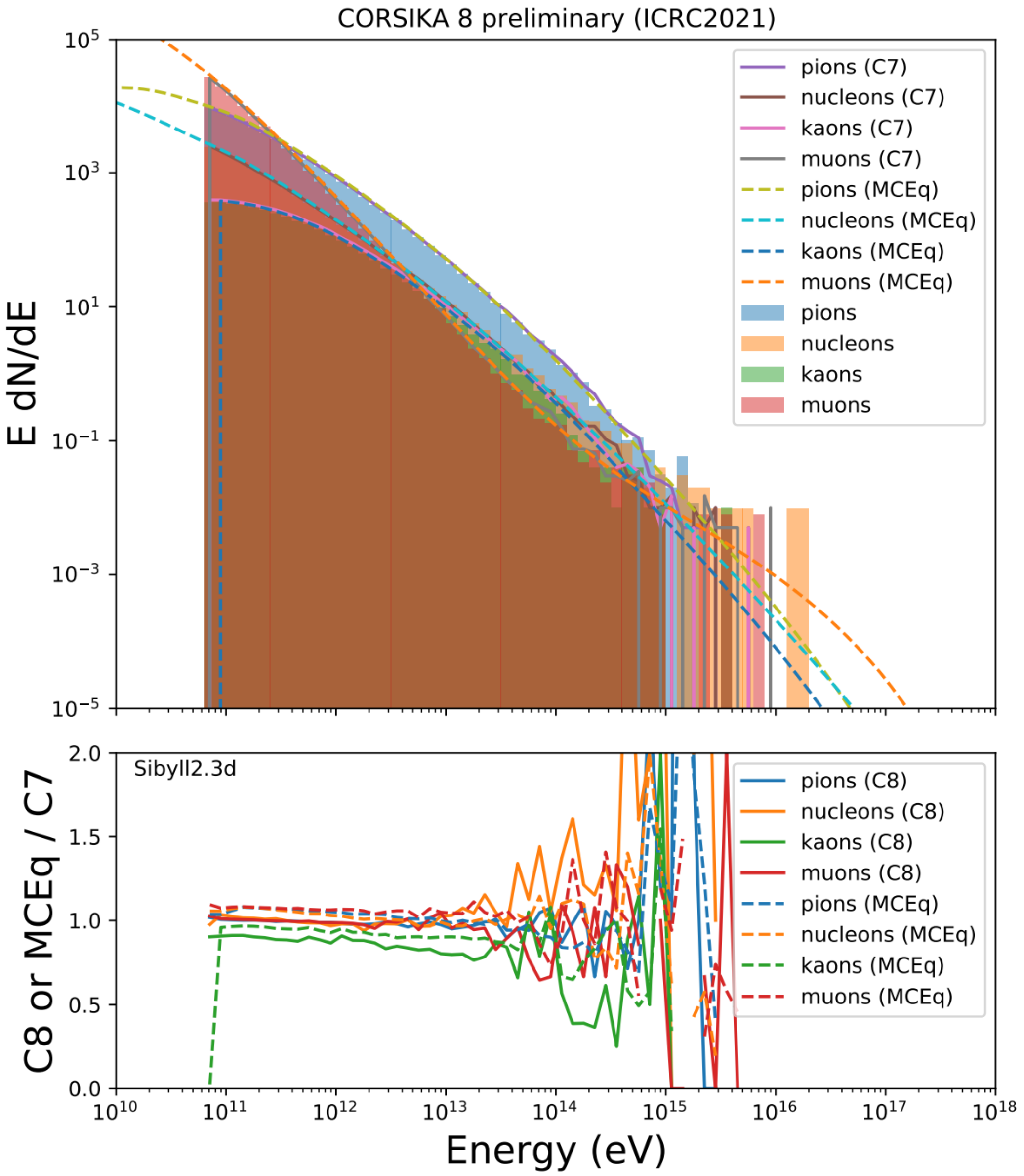
$$X(h_0) = \int_0^{h_0} \mathrm{d}\ell\ \rho_{\mathrm{air}}(\ell)$$

Recent addition by Tania Kozynets **2D-MCEq** (energy + angle), PRD 108 2023, 2306.15263

MCEq vs CORSIKA8 particle spectrum (for average air shower)

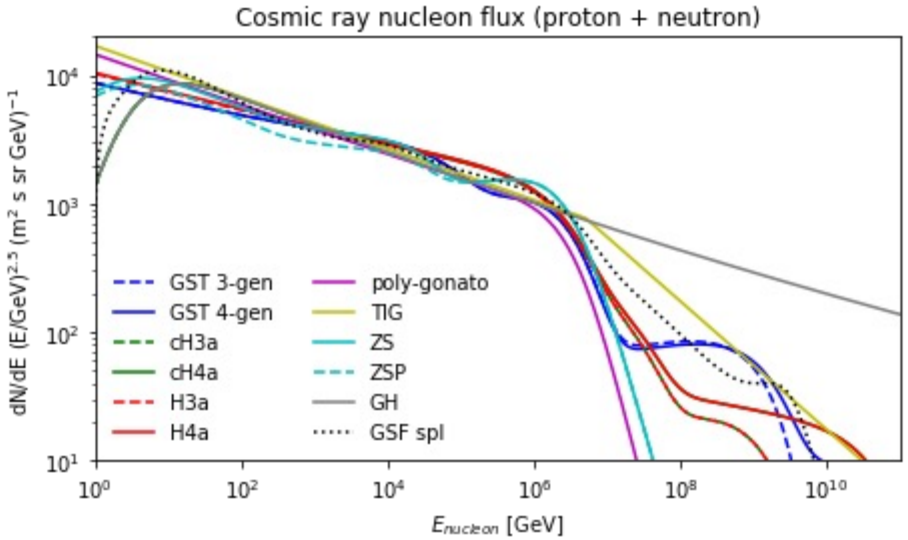
R. Ulrich et al. for C8 Coll.
PoS(ICRC 2021) 474





Available models

Cosmic ray flux models distributed in [an independent crflux module](#).



Hadronic interaction models are:

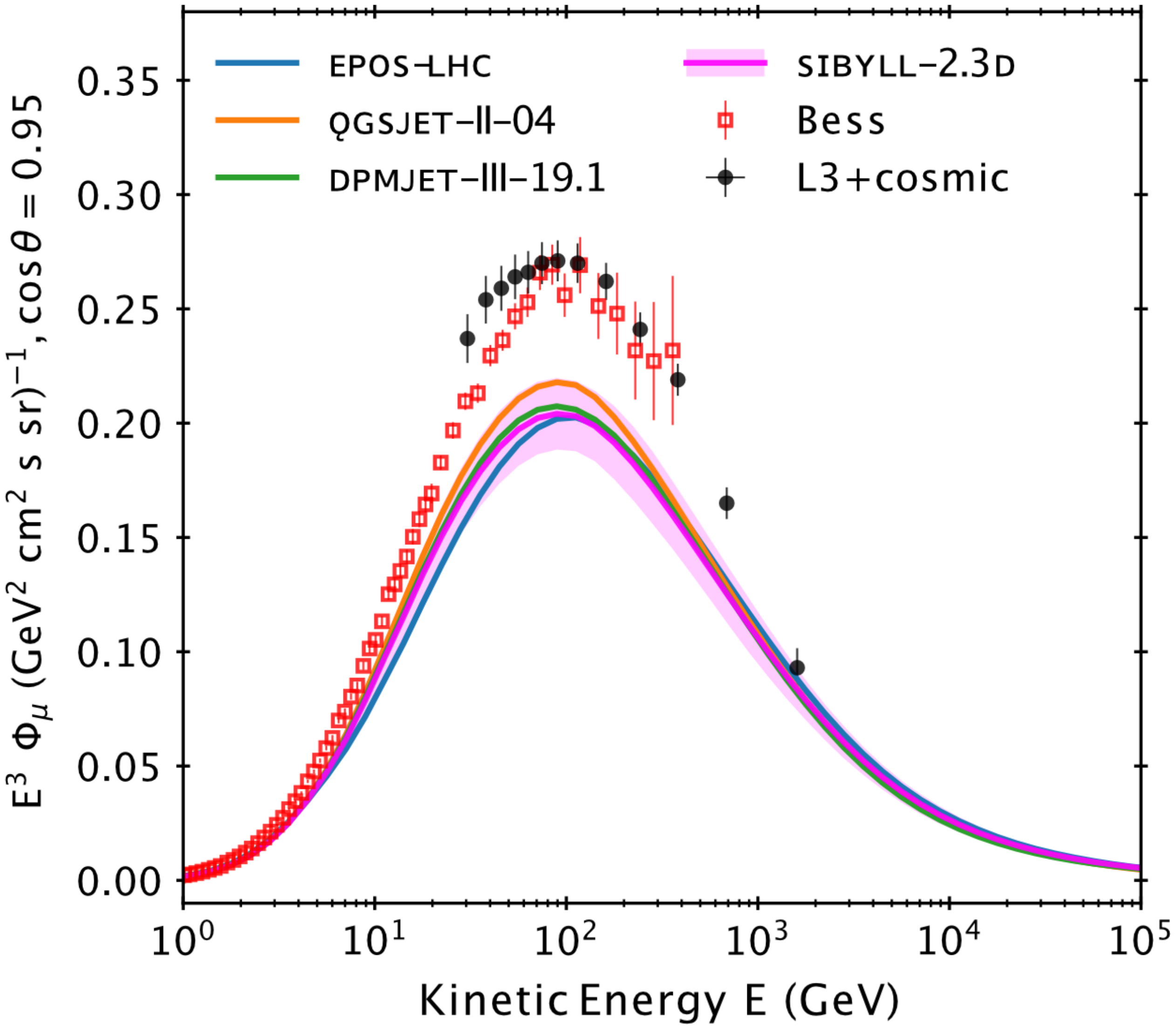
- SIBYLL*
- SIBYLL-2.3c/d + 2.1
- EPOS-LHC
- QGSJet-II-03/-04
- QGSJet-01c
- DPMJET-III-3.0.6
- DPMJET-III-19.1/-3
- FLUKA (work in progress)
- UrQMD (not public)
- Pythia 8 (not public)

Atmosphere models from

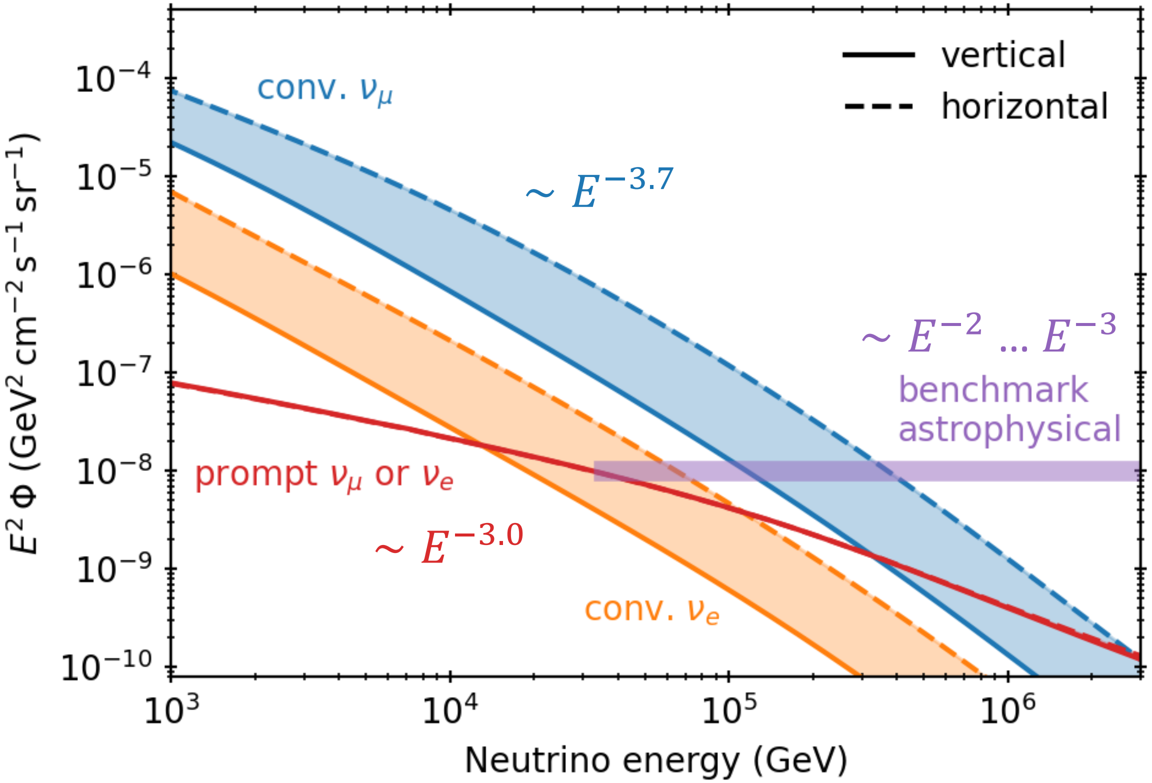
- CORSIKA7 (multiple locations)
- NRLMSISE-00 (global, “static”)
- Some special cases and interface to tabulated atm.

- BSD licensed @ https://github.com/mceq-project/MCEq

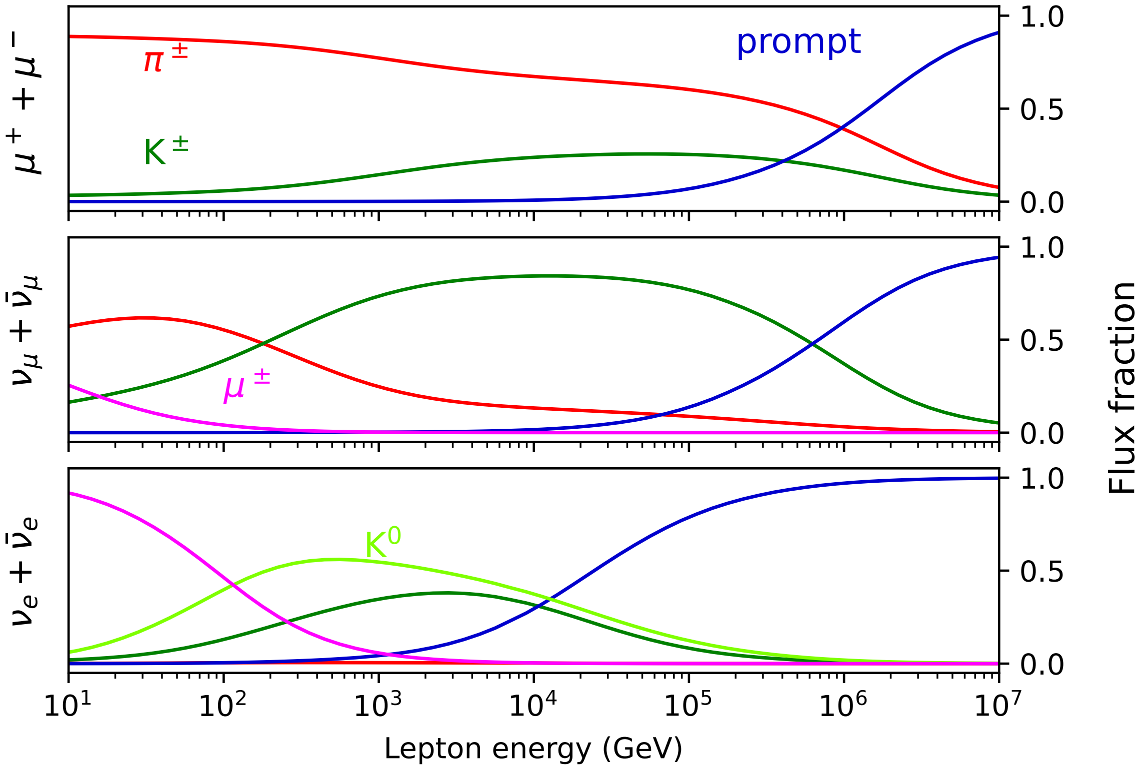
But surface muons never looked great… (known for > 10 years or so)



High energy lepton spectrum



AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019



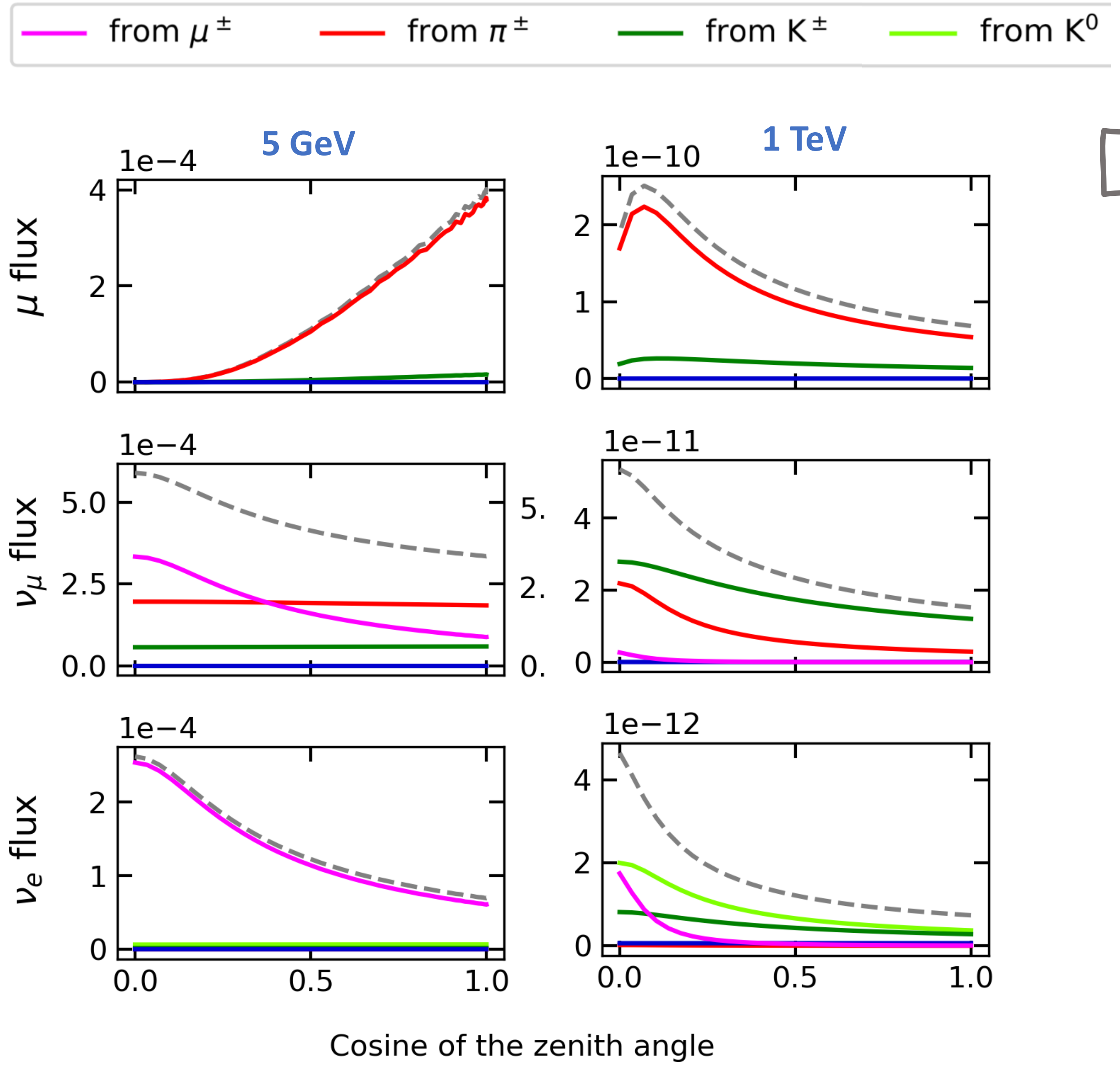
Bands (zenith-enhancement):

- Lower boundary $\cos\theta = 1$, vertical
- Upper boundary $\cos\theta = 0$, horizontal

Different weight of hadrons in lepton production, due to:

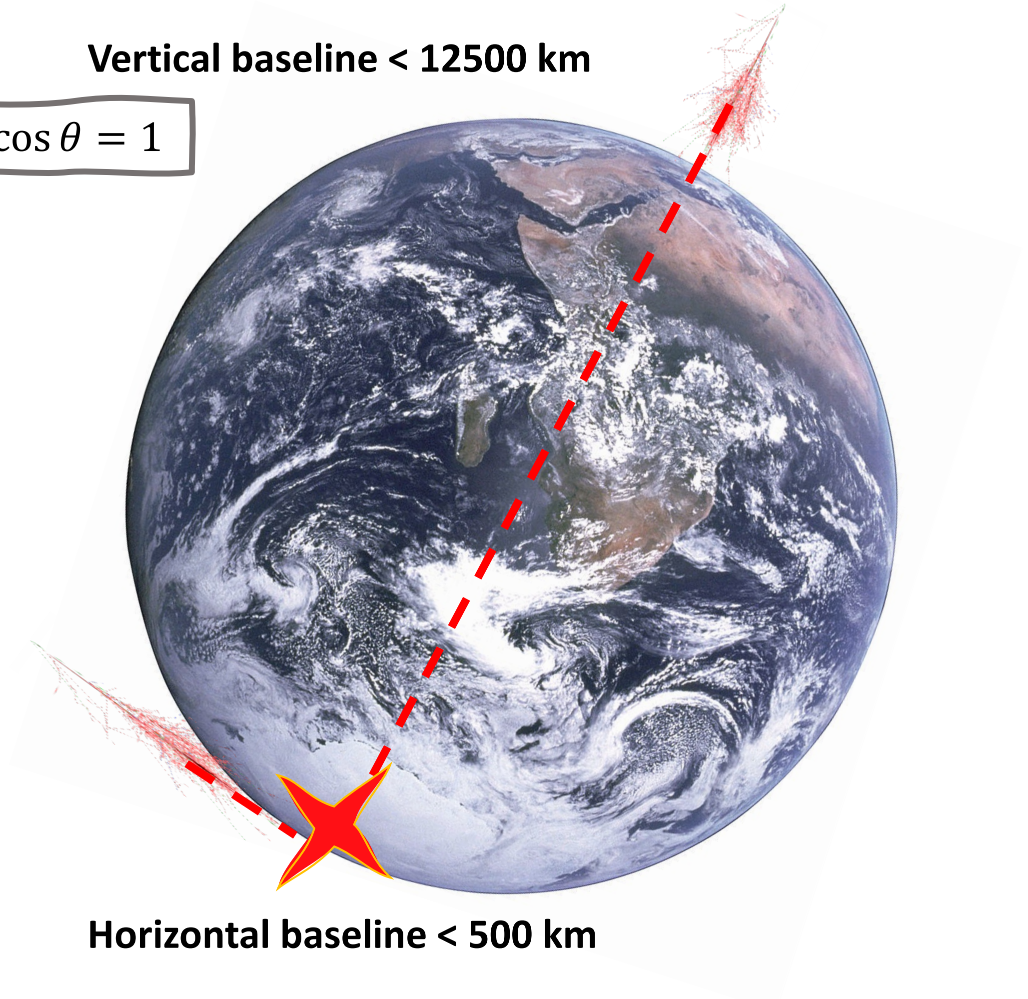
- Hadron production cross sections
- Branching ratio & decay kinematics

Zenith angle dependence at higher-E is sensitive to hadron production



Vertical baseline < 12500 km

vertical: $\cos\theta = 1$



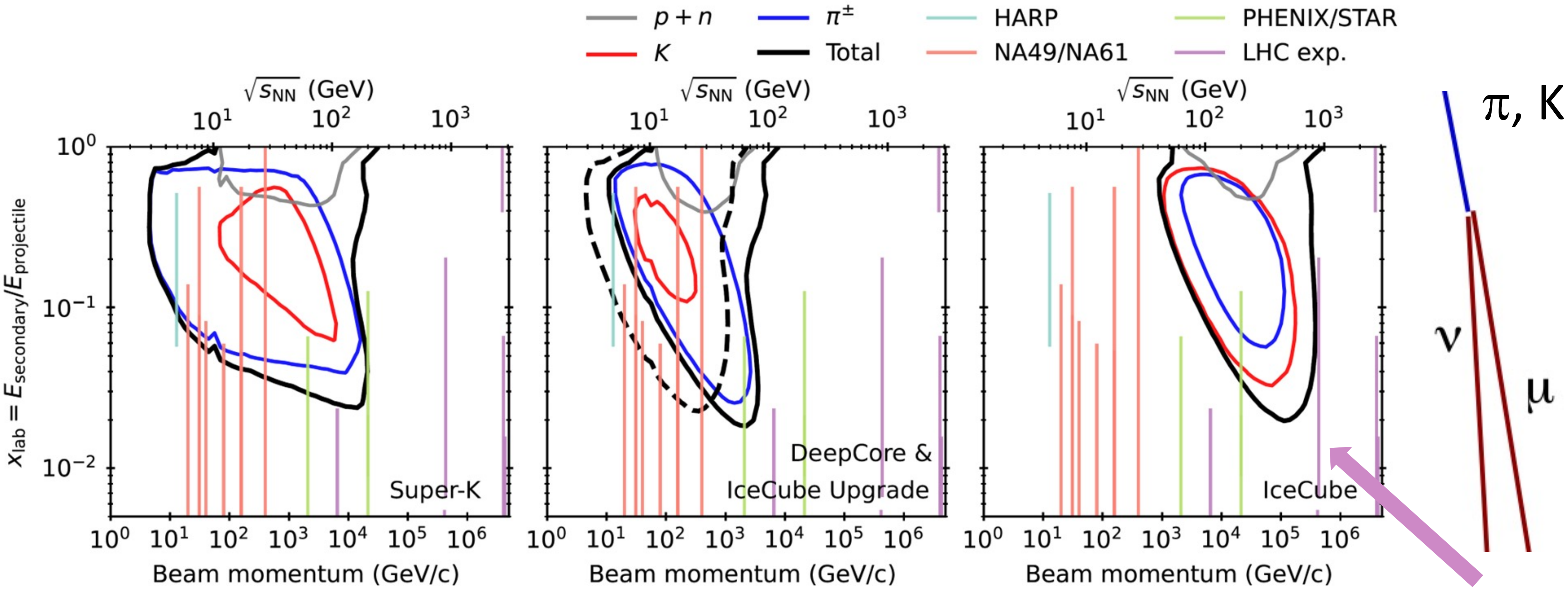
Horizontal baseline < 500 km

horizontal: $\cos\theta = 0$

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019

Hadron production phase space seen by neutrino detectors

AF & M. Huber, arXiv:2205.14766

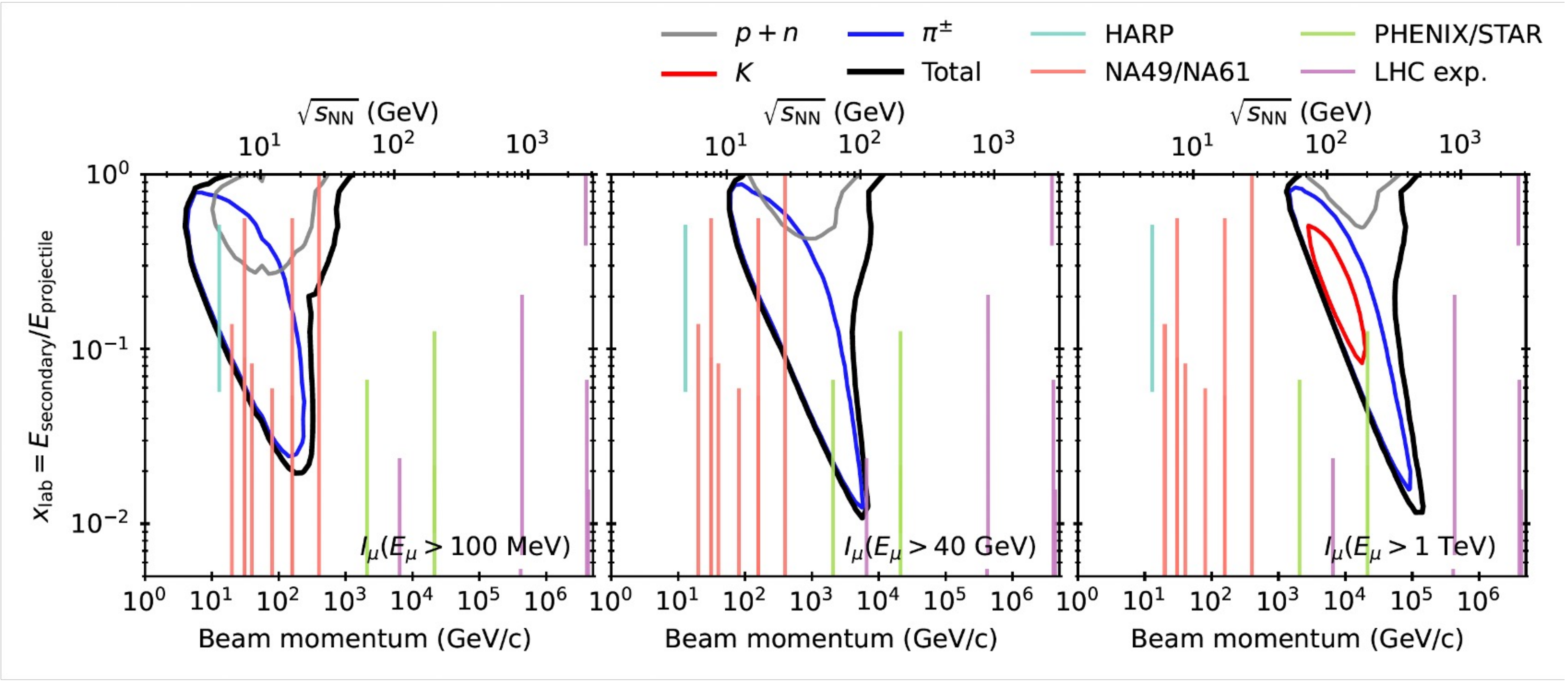


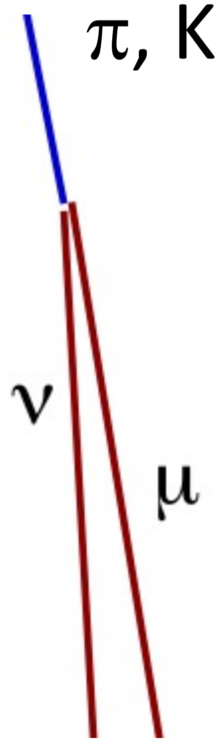
$\sqrt{s} = 900$ GeV

Contours = 90% of neutrino events in full detectors

Related muon production phase space

AF & M. Huber, arXiv:2205.14766



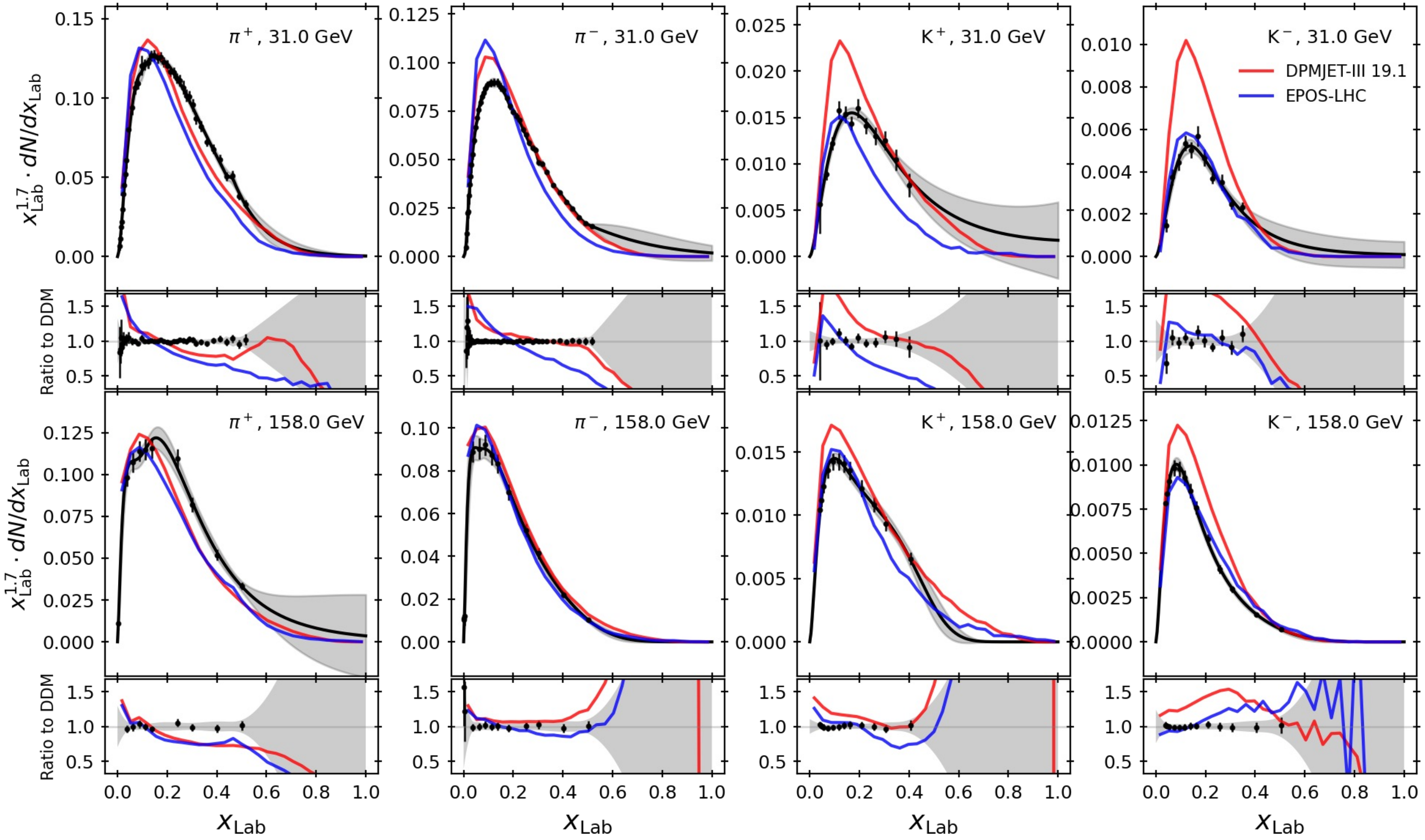


Contours = 90% of integral flux above indicated threshold

Data-driven model (DDM) built in incl. cross sections

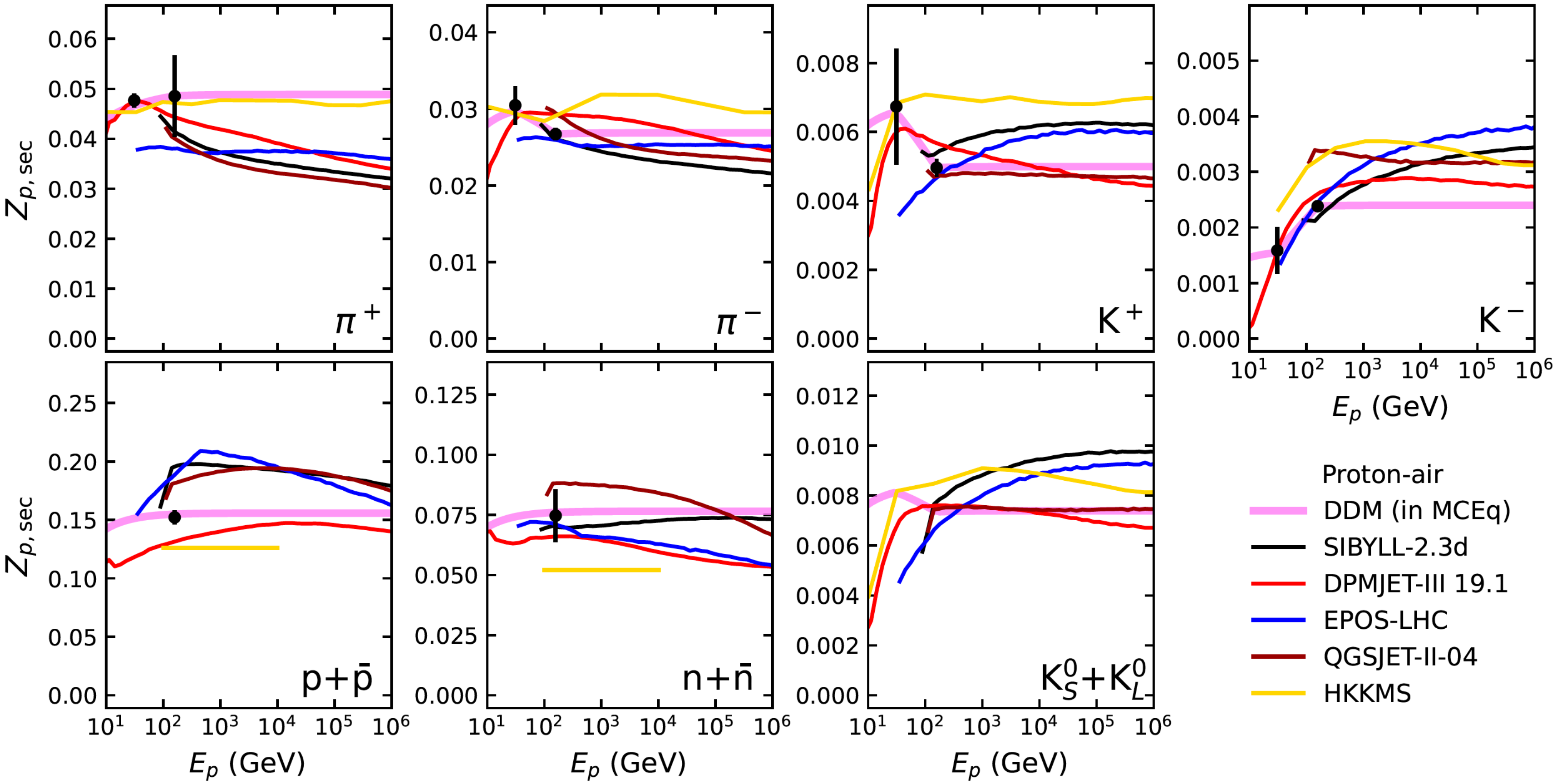
NA49 & NA61 proton-carbon

AF & M. Huber, PRD 106, 2022, arXiv:2205.14766



- **Uncertainties conservatively scale up** in absence of forward data
- K^{+-} data at 158 GeV extrapolated from pp→pC
  - → + 5-7% error from MC
- Carbon to air correction < 1%
- + proton and neutron secondaries , & π^- projectiles (not shown)
- Neutron (and π^+ projectiles) via isospin relations
- K^0 via isospin

Energy inter- and extrapolation



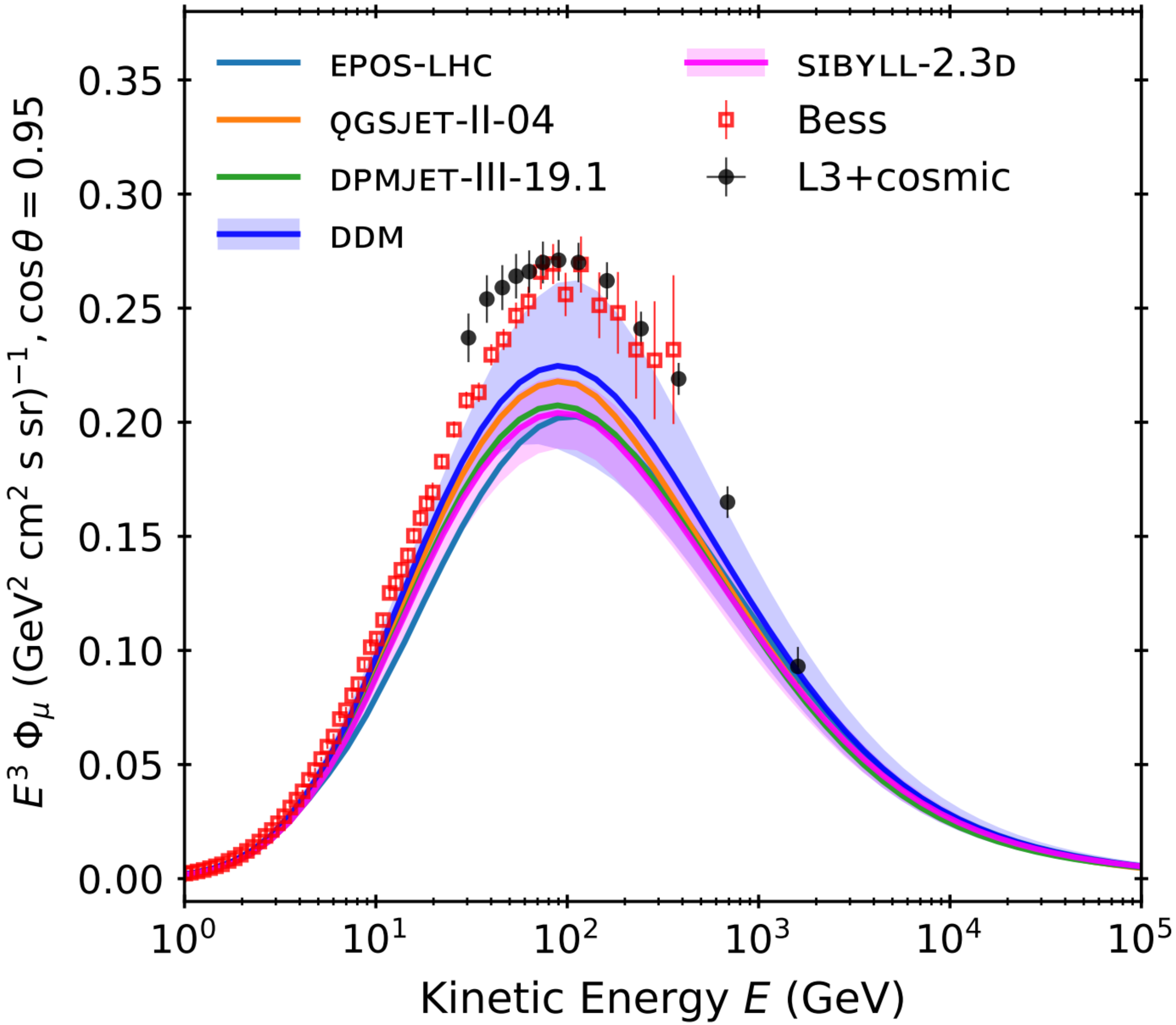
- 1 or 2 cross section "shapes" @ 31 & 158 GeV
- Interpolates linearly in log(E) between those
- Assumes Feynman scaling (shape of longitudinal spectrum constant)
- More points can be added to complicate energy dependence → **daemonflux**

Atm.-flux-relevant phase space
→ Spectrum-weighted moment:

$$Z_{\mathrm{N}h}(E_{\mathrm{N}}) = \int_0^1 \mathrm{d}x_{\mathrm{Lab}}\; x_{\mathrm{Lab}}^{\gamma(E_{\mathrm{N}})-1} \frac{\mathrm{d}N_{\mathrm{N}\to h}}{\mathrm{d}x_{\mathrm{Lab}}}(E_{\mathrm{N}})$$

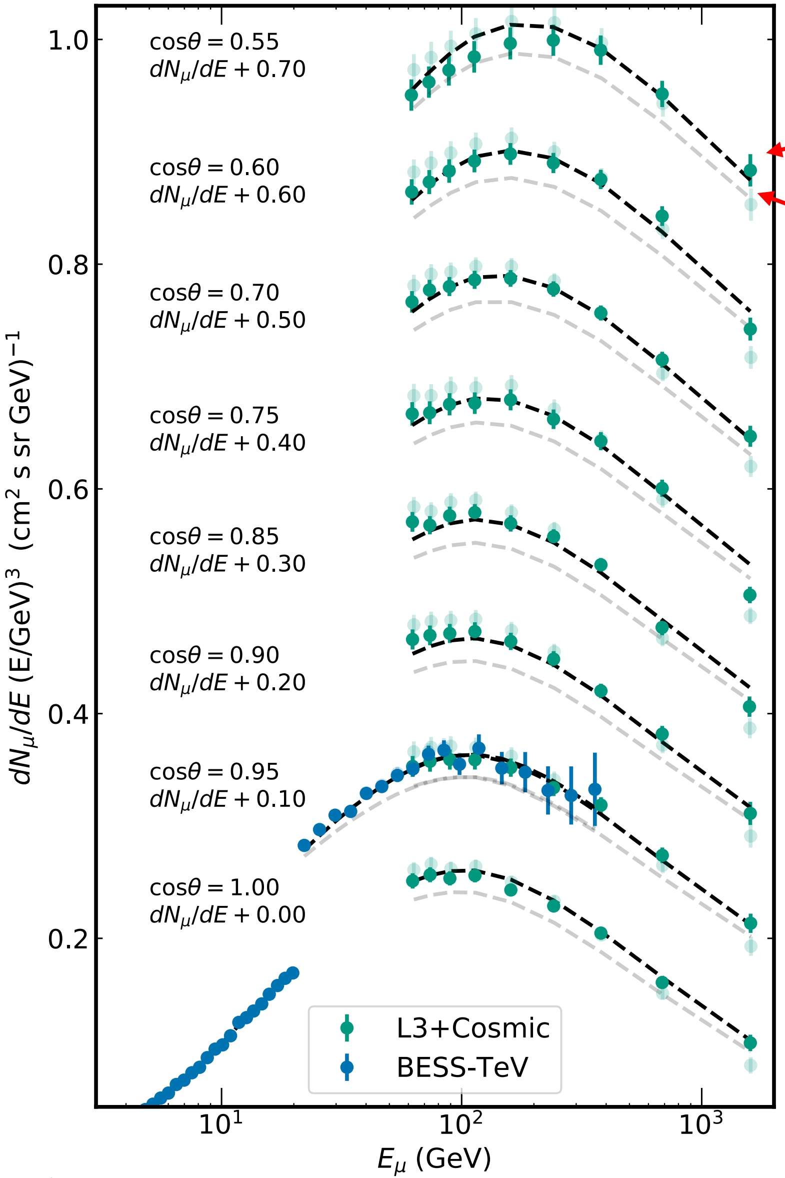
AF & M. Huber, PRD, arXiv:2205.14766
AF, Woodley, Piro, ApJ 2022 arXiv:2109.11559

Atmospheric muon fluxes from DDM + GSF



Resulting muon fluxes and cross-calibrated data (daemonflux)

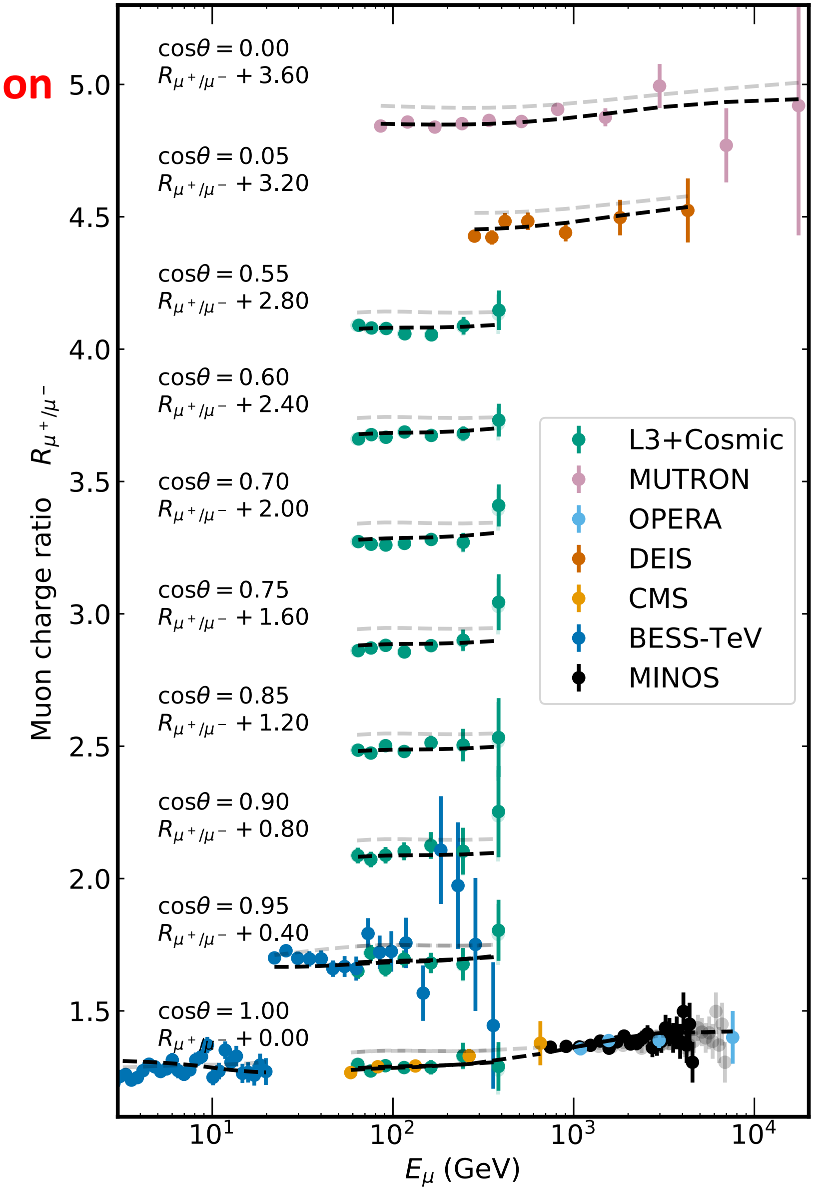
Muon flux



Data w/ syst. correction

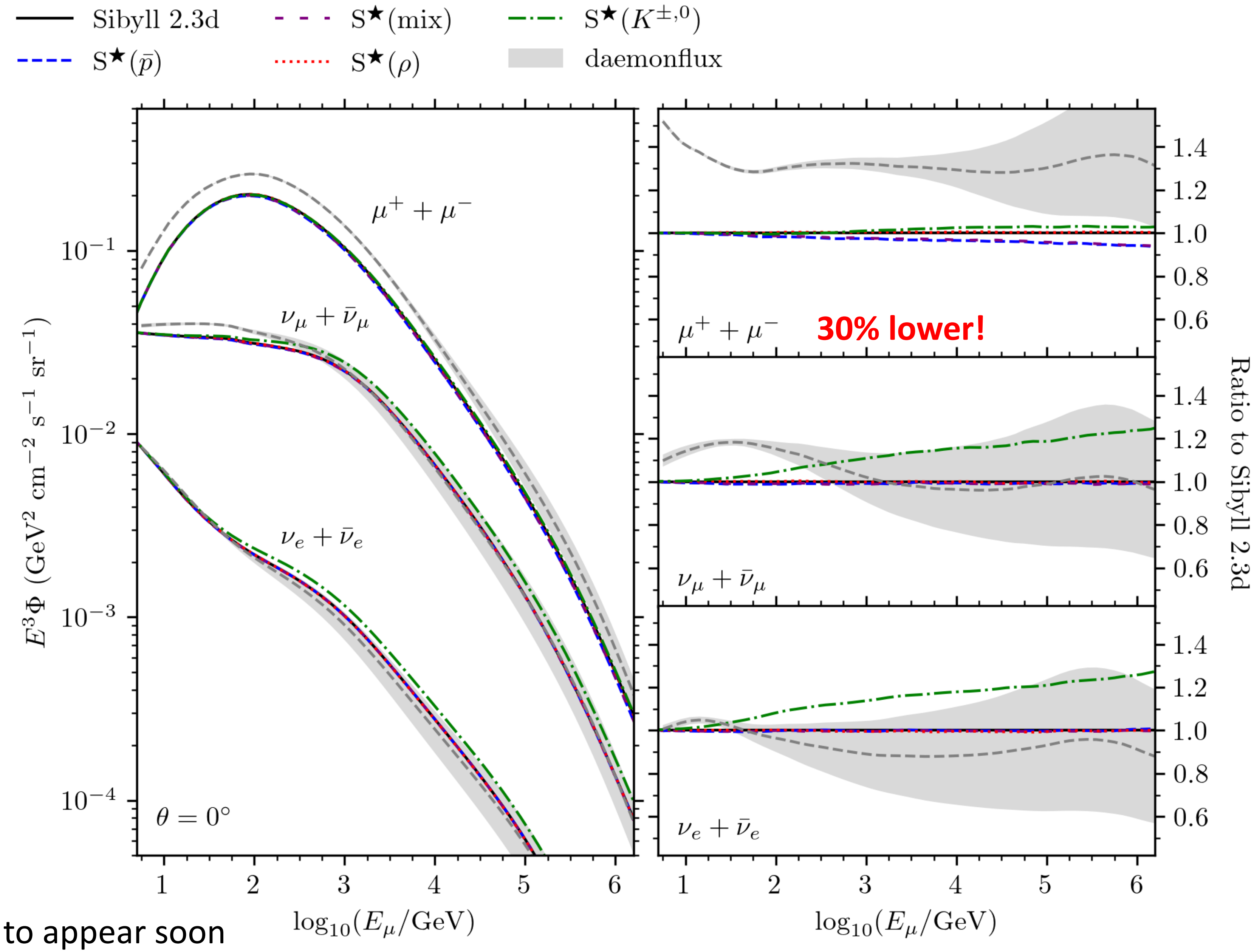
Data w/o syst. correction

Muon charge ratio



J. P. Yanez & AF, arXiv:2303.00022

SIBYLL* vs data-driven muon-calibrated model (daemonflux)



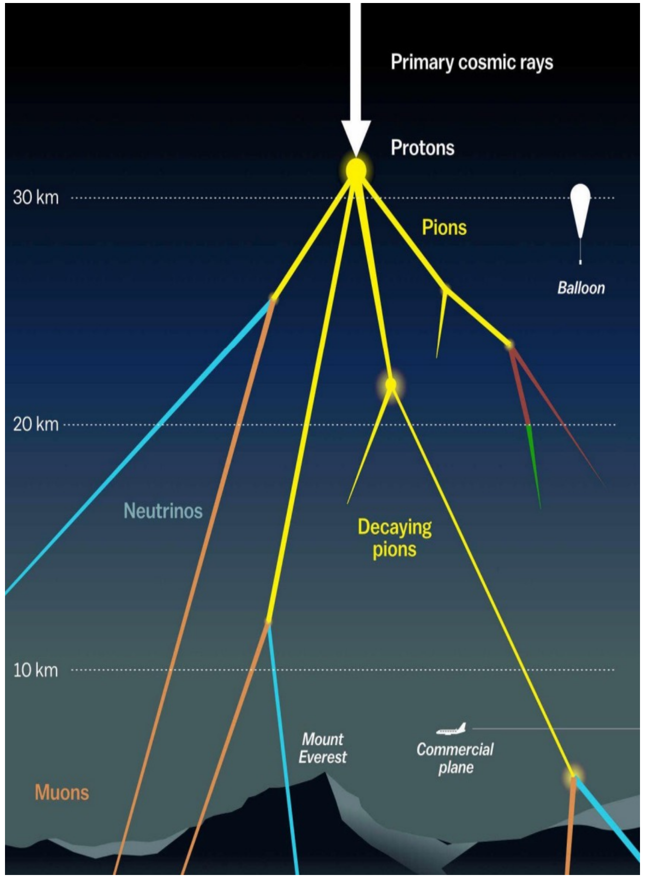
30% lower!

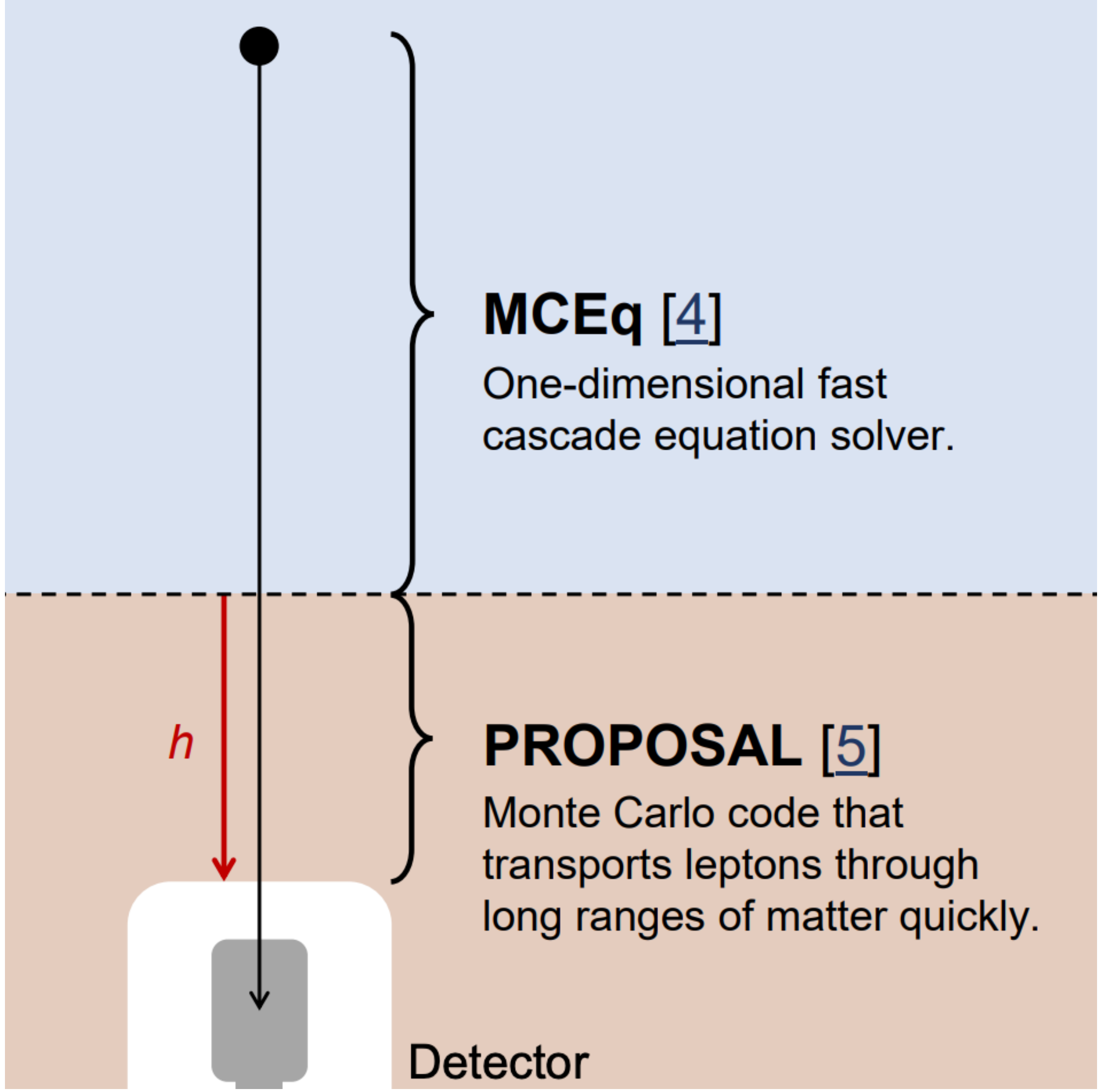
Other IA models in S* ballpark

F. Riehn, AF, R. Engel, to appear soon

High energy constraints from underground μ?

W. Woodley (UofA), TeVPa 2022





MCEq [4]

One-dimensional fast cascade equation solver.

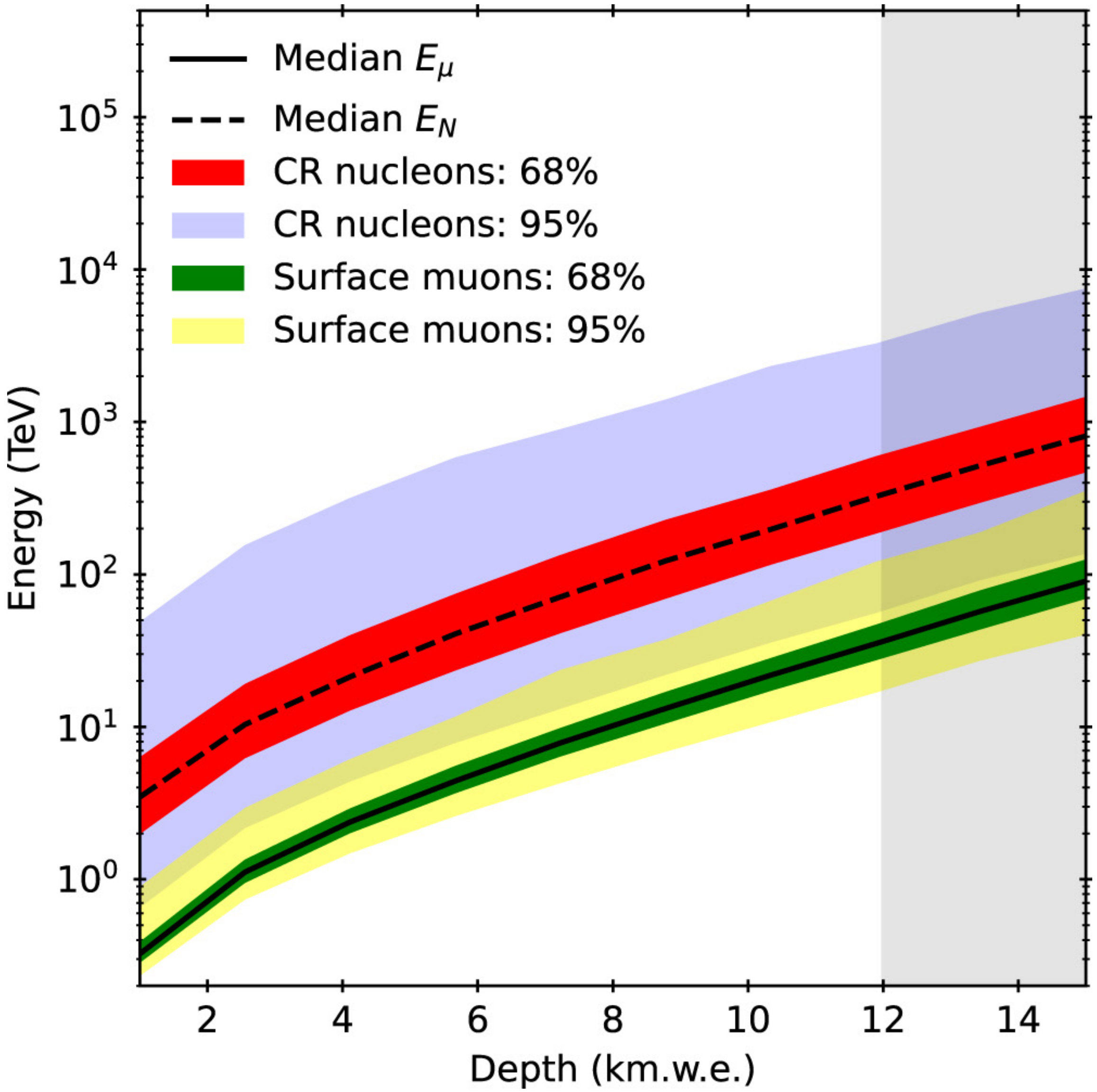
PROPOSAL [5]

Monte Carlo code that transports leptons through long ranges of matter quickly.

AF, **W. Woodley**, M.-C. Piro, *ApJ* **928** 27 (2022)

W. Woodley, TeVPa 2022 and Woodley, AF, Piro in prep.

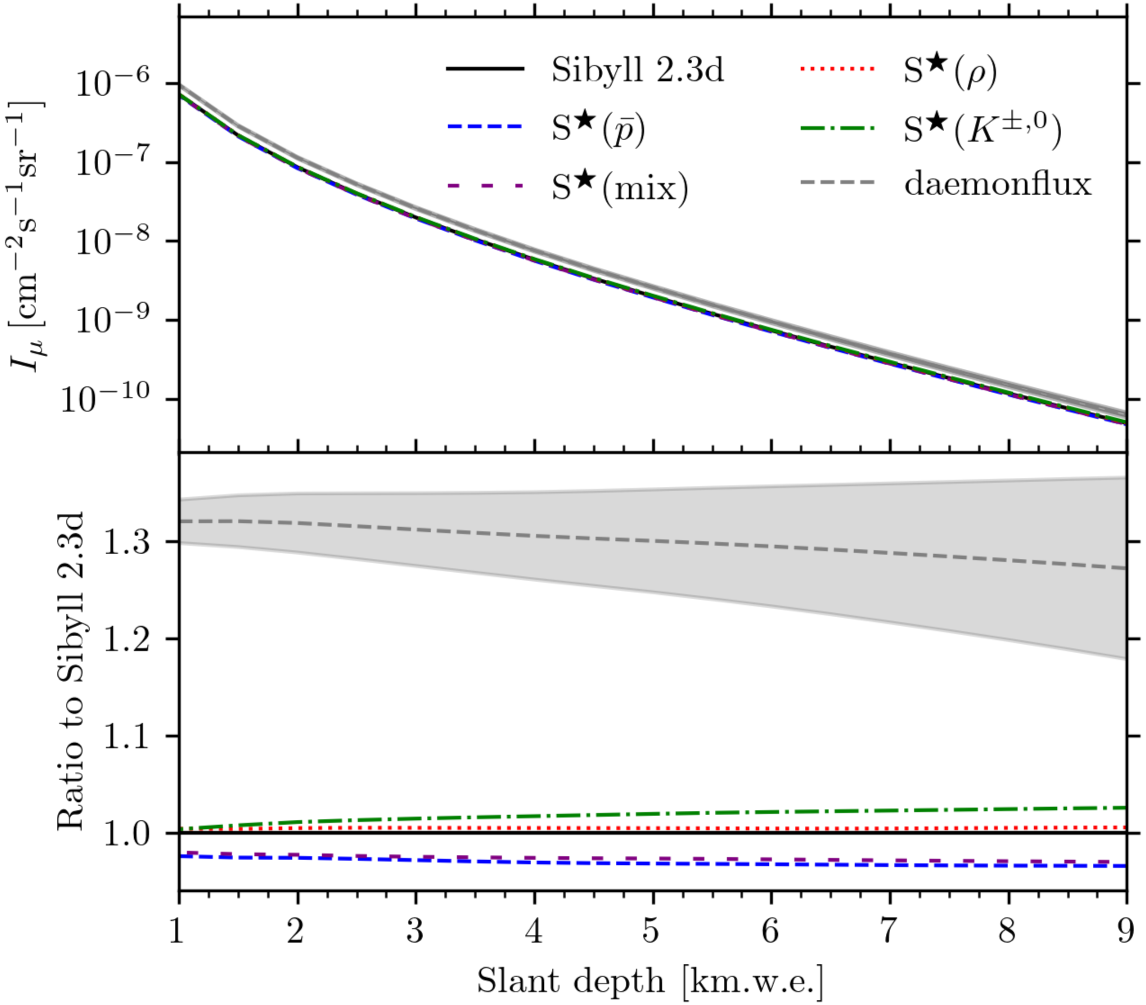
Relation of depth to surface and CR energy

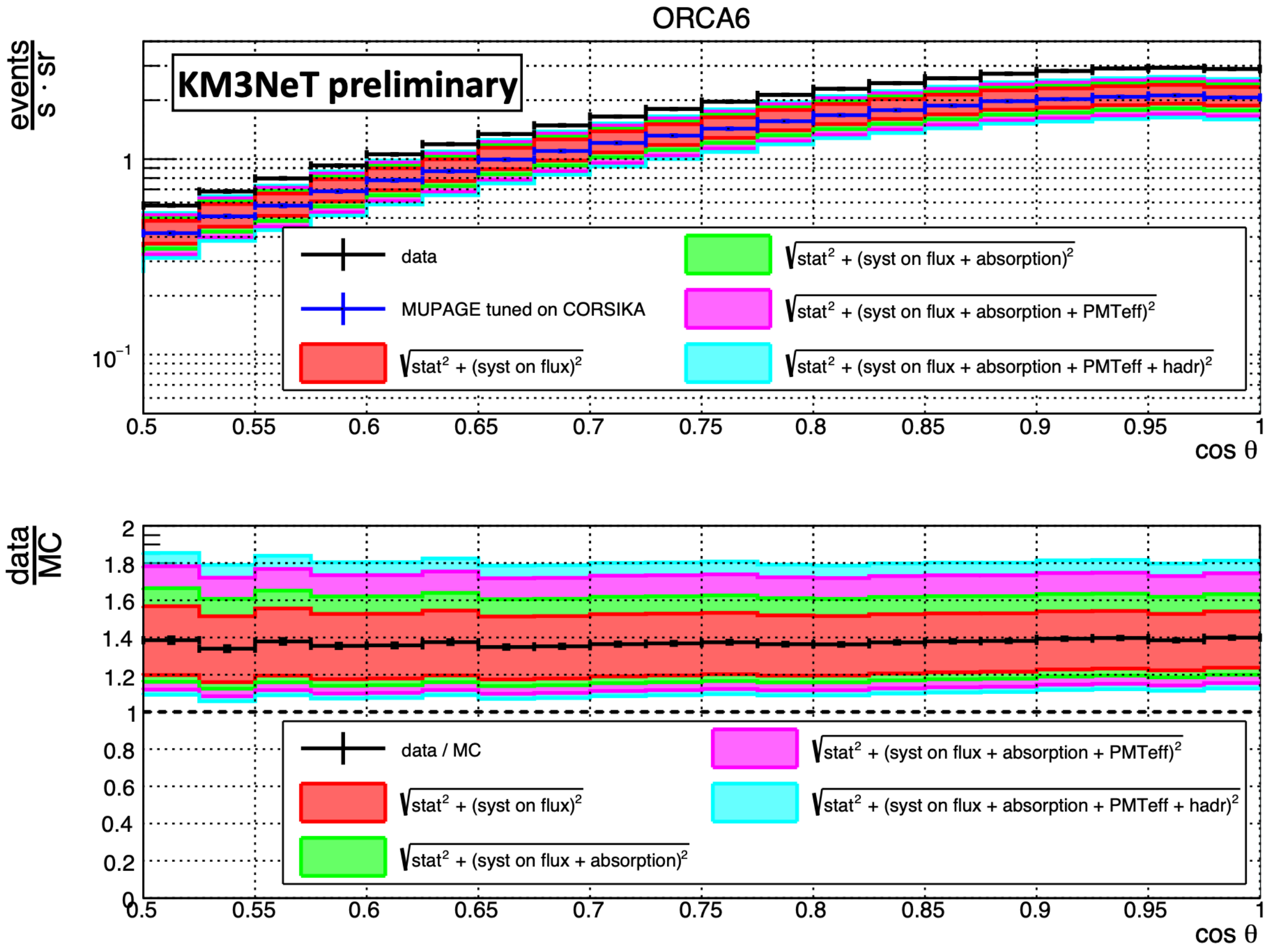


AF, W. Woodley, M.-C. Piro, *ApJ* **928** 27 (2022)

Daemonflux vs models underground/-water

A. Romanov et al. (KM3NeT), PoS(ICRC2023) 338

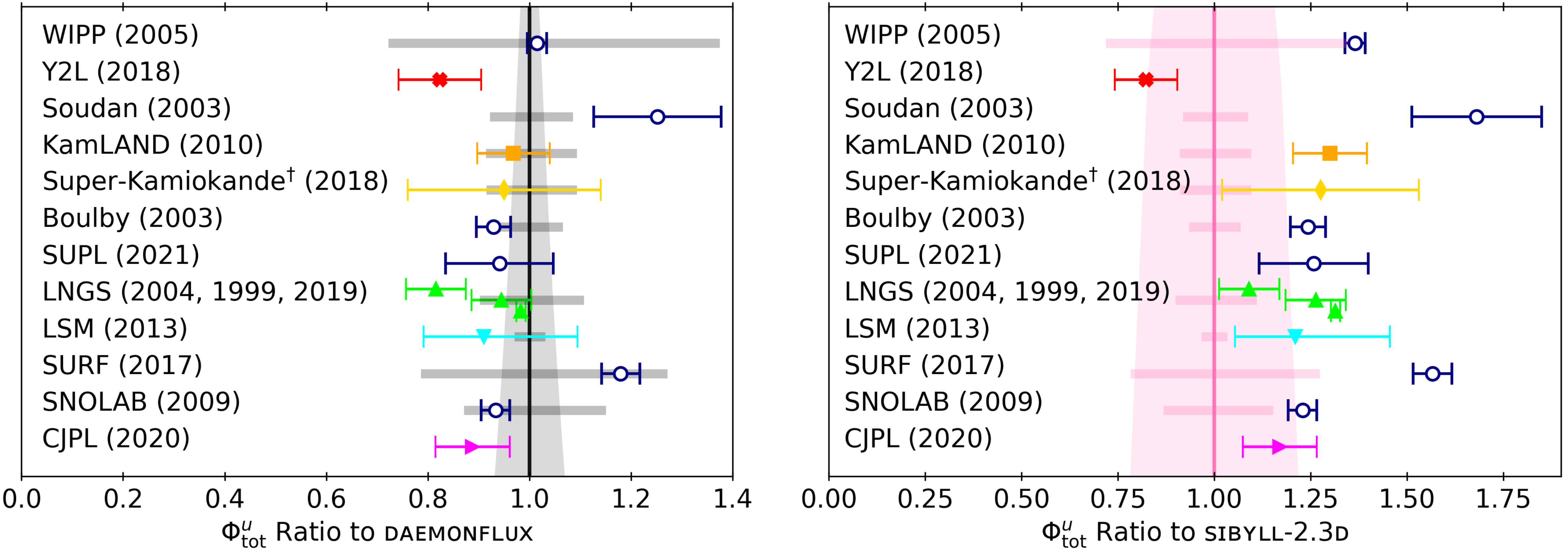




F. Riehn, AF, R. Engel, to appear soon

> 30% discrepancy!

Total muon fluxes underground: “simple” measurement



Woodley, AF, Piro, shown at PoS(ICRC2023) 338, paper to appear soon

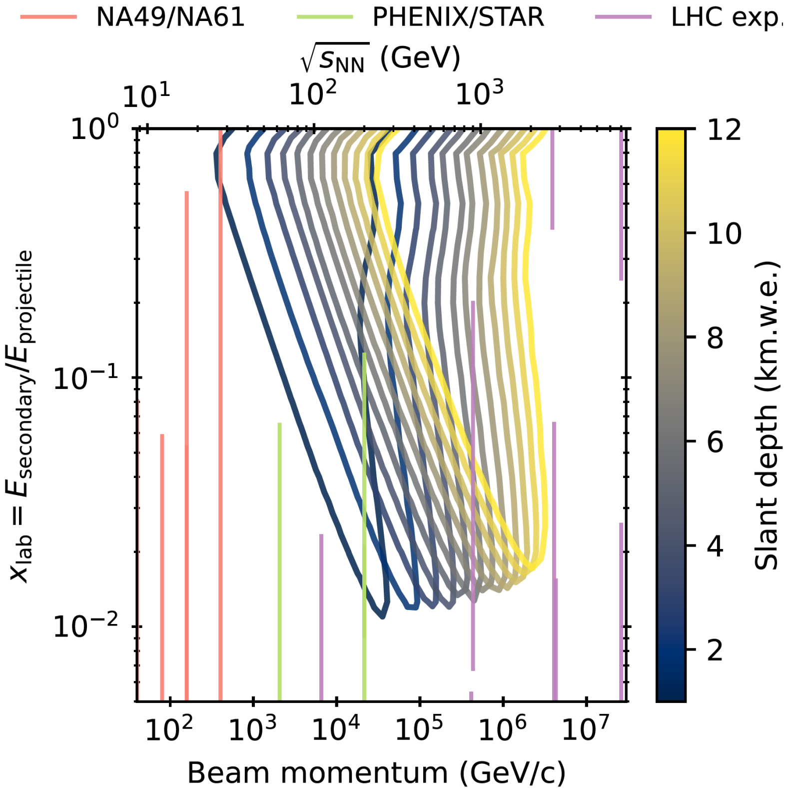
- Measurement almost model independent
- Calculations difficult (chem. Rock composition, density, overburden data)

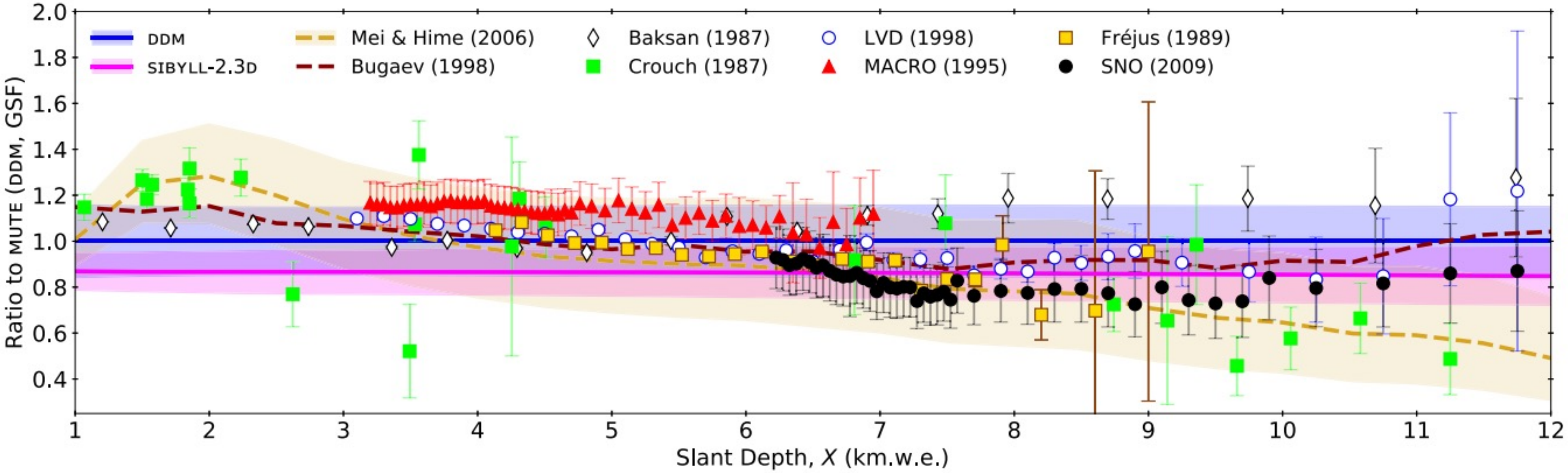
Summary

- MCEq is a generic tool, validated against data and other simulations
- Atm. Leptons are a different channel to study very forward hadronic interactions (mostly p-air)
- “Differences“ seen in comparisons with muon data at the surface and underground
- Validation/calibration via muon surface fluxes very challenging if performed rigorously!
  (old data and docs)
- **Models 30-35% lower than muon data above a few tens of GeV**
- Discrepancy in neutrinos (sensitive to kaon production) experimentally not established
- Origin of discrepancies different from the muon excess in air showers (SIBYLL*)
- Current work is on understanding data

Underground data constraining if systematics understood

AF, **W. Woodley**, M.-C. Piro, *ApJ* **928** 27 (2022)

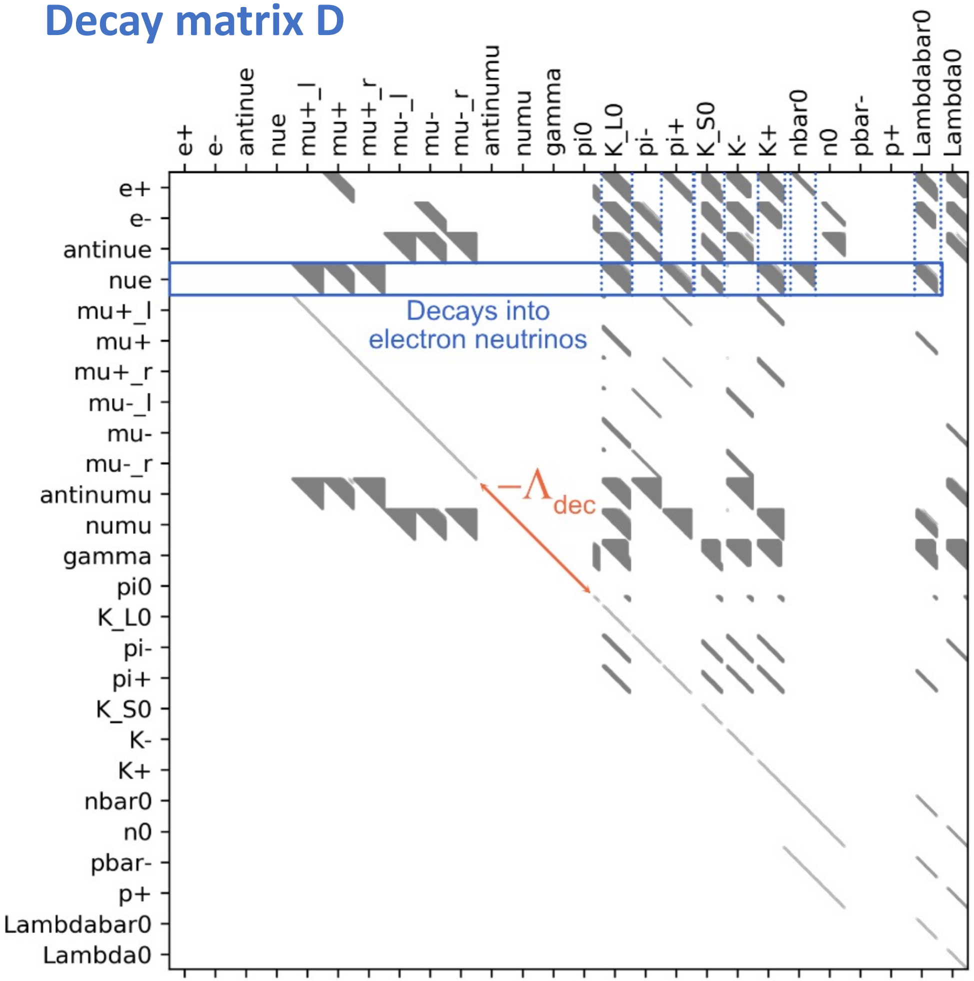




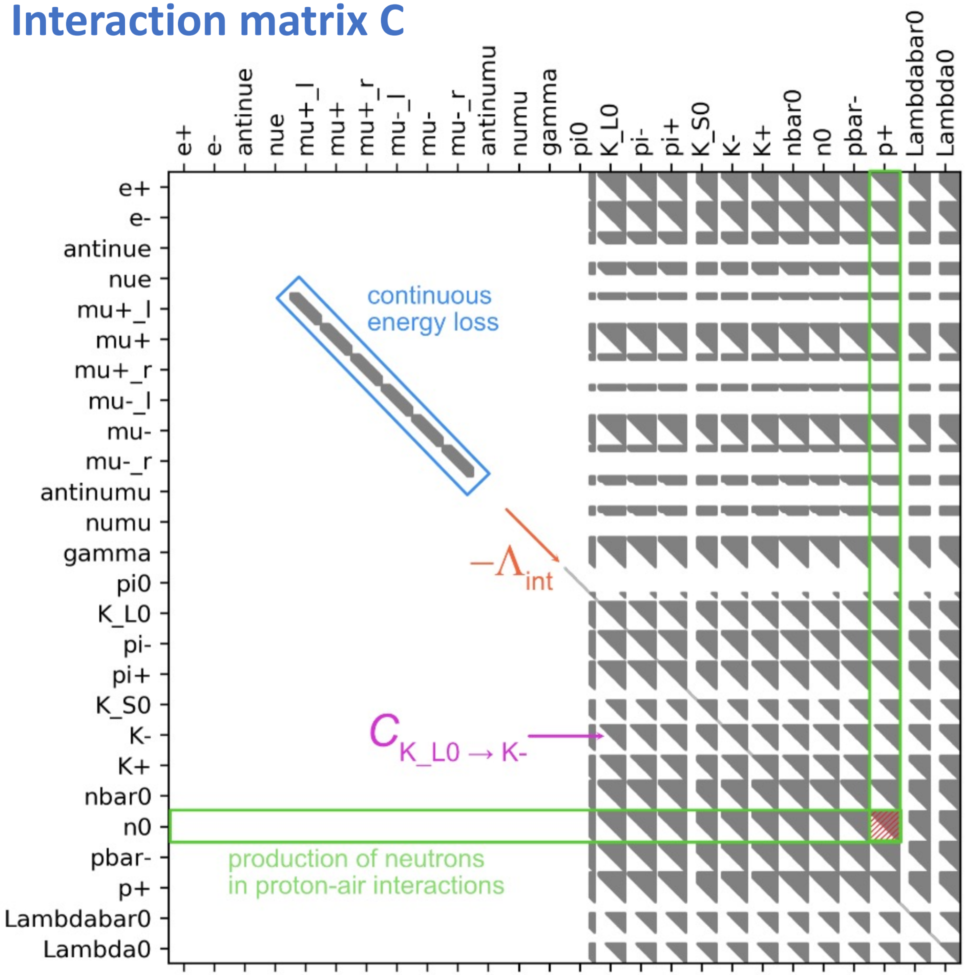
- Vertical equivalent rate, total underground muon rate, 2D distributions and seasonal variations for labs under mountains (paper in prep for ICRC)
- Underground muon charge ratio (not unfolded) (MINOS?)
- New fast code by William Woodley (MUTE) https://github.com/wjwoodley/mute
- Attempt combined fit with surface muons → nail down high energy uncertainties
- **Challenge: survey experimental data with explicit systematic uncertainties**

Sparse matrix structure

Decay matrix D



Interaction matrix C



matrices are sparse → high performance

MCEq: Matrix Cascade Equations

A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor
PoS ICRC 2015, 1129 (2015), EPJ Web Conf. 99, 08001 (2015) and EPJ Web Conf. 116, 11010 (2016)

$$
\begin{aligned}
\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = &-\frac{\Phi_h(E,X)}{\lambda_{\mathrm{int},h}(E)} \\
&-\frac{\Phi_h(E,X)}{\lambda_{\mathrm{dec},h}(E,X)} \\
&-\frac{\partial}{\partial E}(\mu(E)\Phi_h(E,X)) \\
&+\sum_{\ell}\int_{E}^{\infty}\mathrm{d}E_\ell\,\frac{\mathrm{d}N_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E}\frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{int},l}(E_\ell)} \\
&+\sum_{\ell}\int_{E}^{\infty}\mathrm{d}E_\ell\,\frac{\mathrm{d}N^{\mathrm{dec}}_{\ell(E_\ell)\to h(E)}}{\mathrm{d}E}\frac{\Phi_\ell(E_\ell,X)}{\lambda_{\mathrm{dec},l}(E_\ell,X)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\Phi^h_{E_i}}{\mathrm{d}X} = &-\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{int},E_i}} \\
&-\frac{\Phi^h_{E_i}}{\lambda^h_{\mathrm{dec},E_i}(X)} \\
&-\vec{\nabla}_i(\mu^h_{E_i}\Phi^h_{E_i}) \\
&+\sum_{E_k\geq E_i}^{E_N}\sum_{\ell}\frac{c_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{int},E_k}}\Phi^\ell_{E_k} \\
&+\sum_{E_k\geq E_i}^{E_N}\sum_{\ell}\frac{\mathrm{d}_{\ell(E_k)\to h(E_i)}}{\lambda^\ell_{\mathrm{dec},E_k}(X)}\Phi^\ell_{E_k}
\end{aligned}
$$

State (or flux) vector

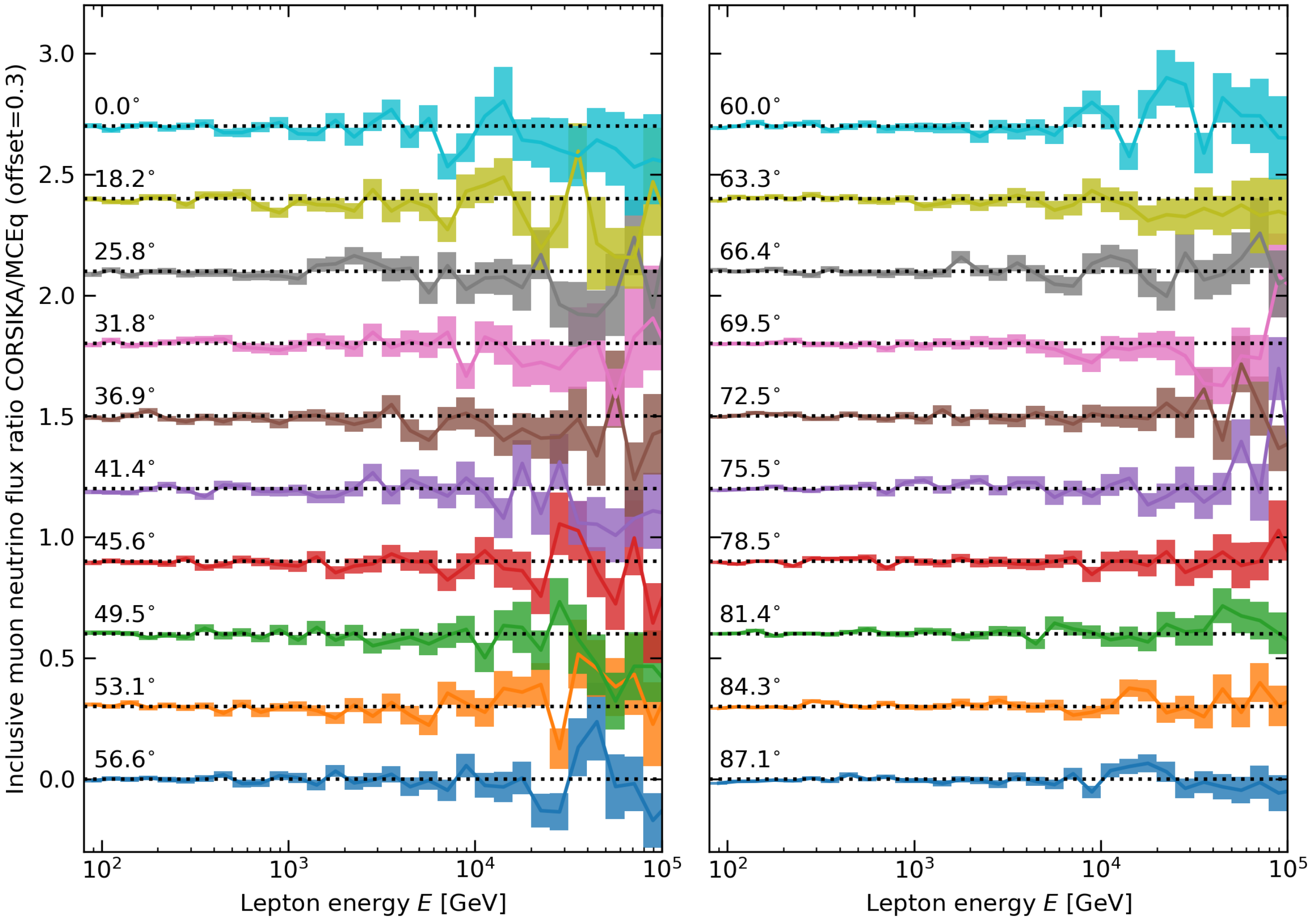
$$\vec{\Phi} = \left(\vec{\Phi}^{\mathrm{p}} \quad \vec{\Phi}^{\mathrm{n}} \quad \vec{\Phi}^{\pi^+} \quad \dots \quad \vec{\Phi}^{\bar{\nu}_\mu} \quad \dots\right)^T$$

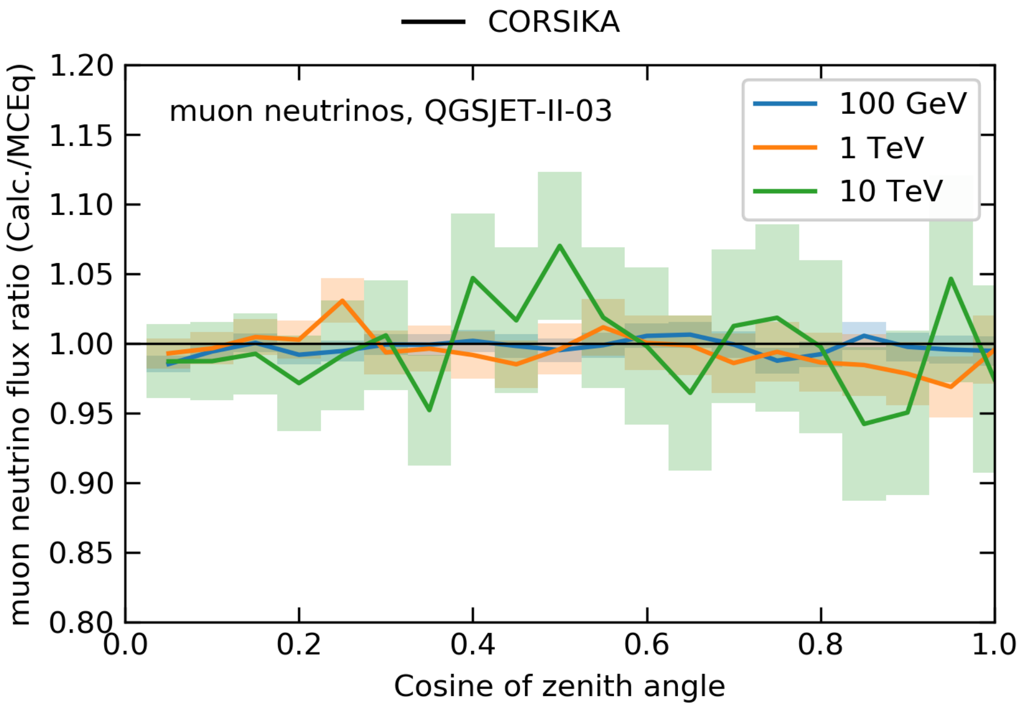
$$\vec{\Phi}^{\mathrm{p}} = \left(\Phi^{\mathrm{p}}_{E_0} \quad \Phi^{\mathrm{p}}_{E_1} \quad \cdots \quad \Phi^{\mathrm{p}}_{E_N}\right)^T$$

“Matrix form”

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}X}\vec{\Phi} = &-\vec{\nabla}_E(\mathrm{diag}(\vec{\mu})\vec{\Phi}) + (-\mathbf{1}+\mathbf{C})\mathbf{\Lambda}_{\mathrm{int}}\vec{\Phi} \\
&+\frac{1}{\rho(X)}(-\mathbf{1}+\mathbf{D})\mathbf{\Lambda}_{\mathrm{dec}}\vec{\Phi}
\end{aligned}
$$

MCEq vs CORSIKA7 inclusive spectra

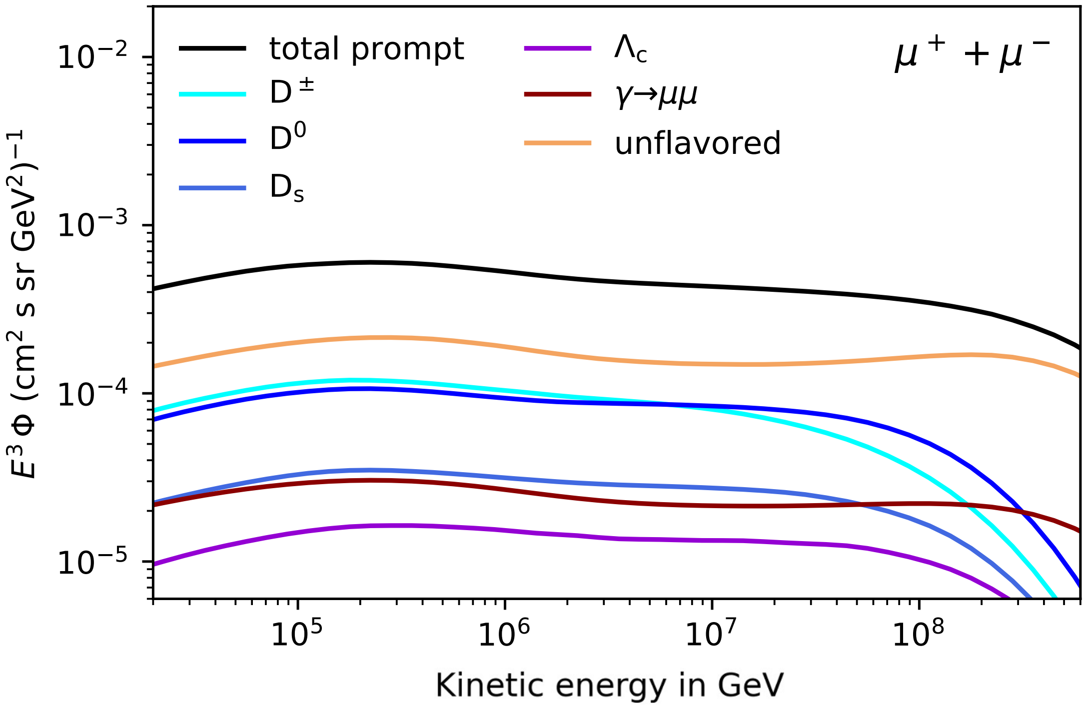

Inclusive muon neutrino flux ratio CORSIKA/MCEQ. QGSJET-II-03 + H3a.



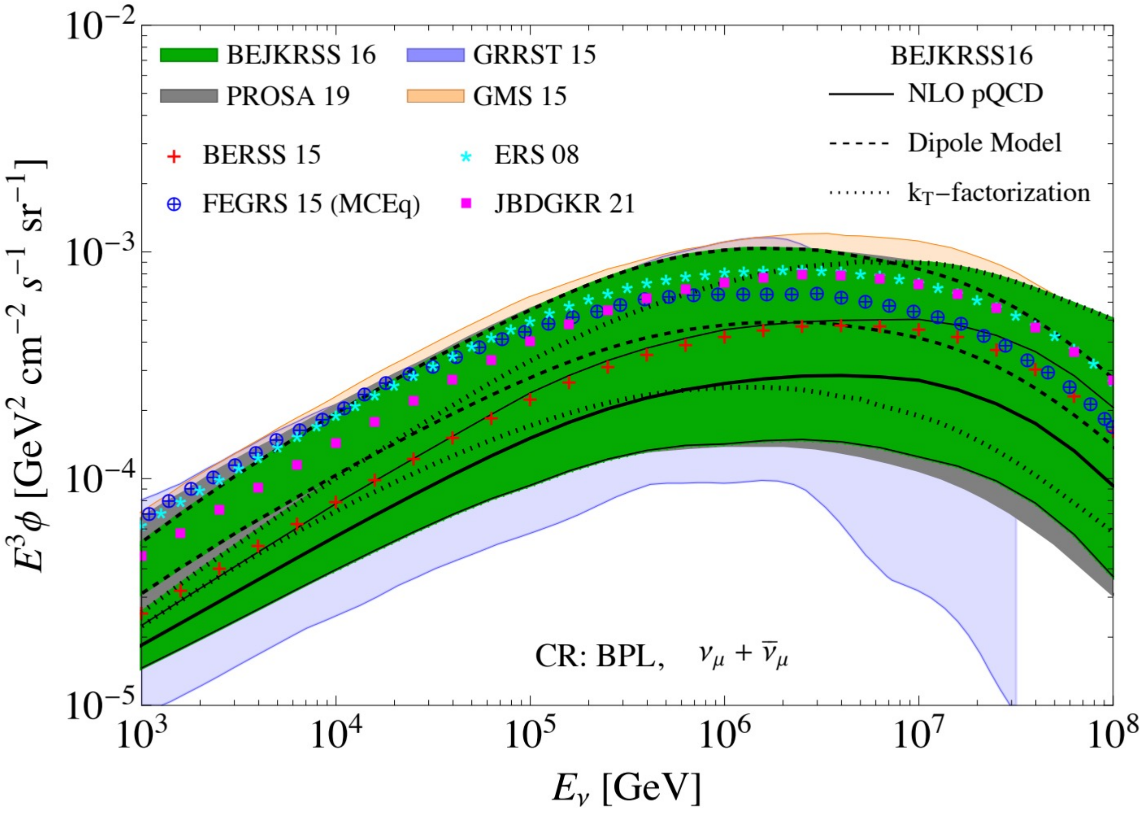
CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012

Above 100 TeV: territory of the (undiscovered) prompt muons and neutrinos

AF, F. Riehn, R. Engel, T.K. Gaisser, T. Stanev, PRD 100 2019



Forward Physics Facility Snowmass arXiv: 2203.05090

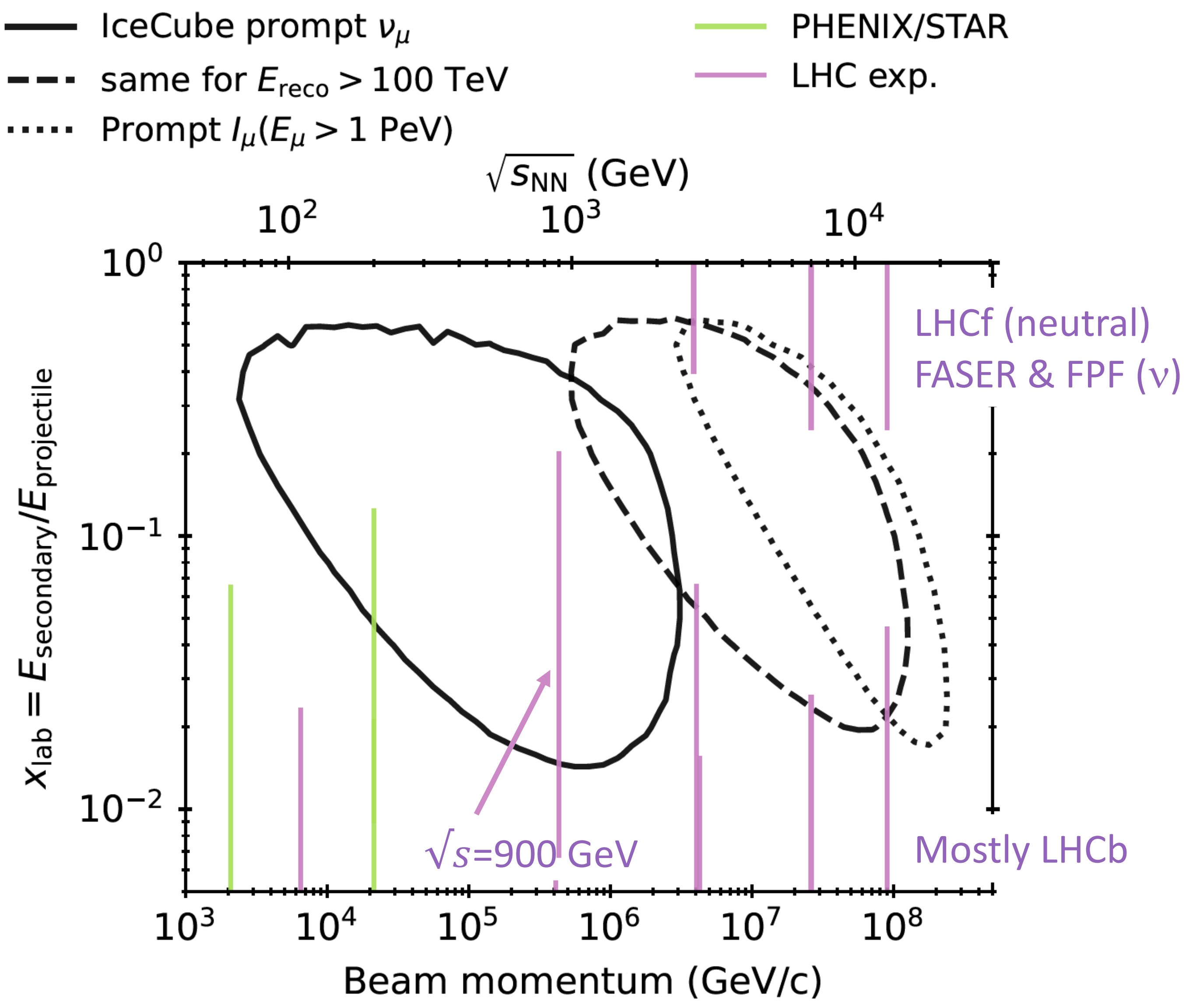


Prompt muons more production channels than prompt neutrinos:

- Rare decays of unflavored mesons *e.g.,* $\eta \to \mu^+\mu^-$
- EM pair production $\gamma \to \mu^+\mu^-$

- Large uncertainties from pQCD
- pQCD might be incomplete (intrinsic charm)
- The fragmentation (c $\to D$) function is a choice

Charm production cross section inaccessible to present-day colliders



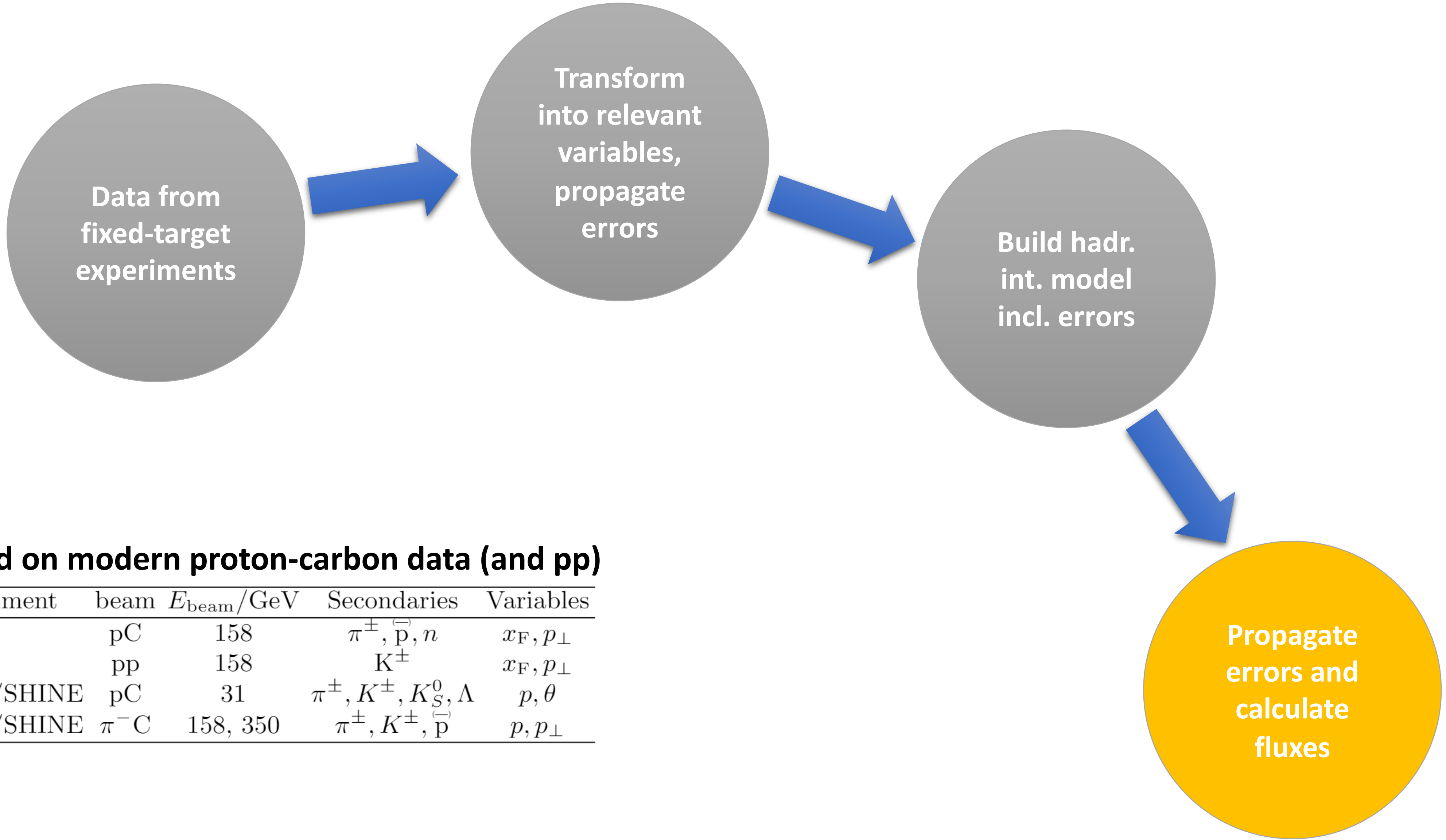
- Each line represents a collider running at fixed $\sqrt{s}$
- Gap in x between LHC coverage is due to the beam pipe
- Detectors need particle ID capability & sufficient luminosity
- Indirect constraints from new forward detectors like FASER and the proposed FPF (see 2203.05090)
- New insights expected from proton-oxygen collisions in Run3

Neutrino contours: 90% events in IceCube

Muon contours: 90% of integral flux

AF & M. Huber, arXiv:2205.14766

Data-Driven Hadronic Interaction Model (DDM)


Data from fixed-target experiments

Transform into relevant variables, propagate errors

Build hadr. int. model incl. errors

Propagate errors and calculate fluxes

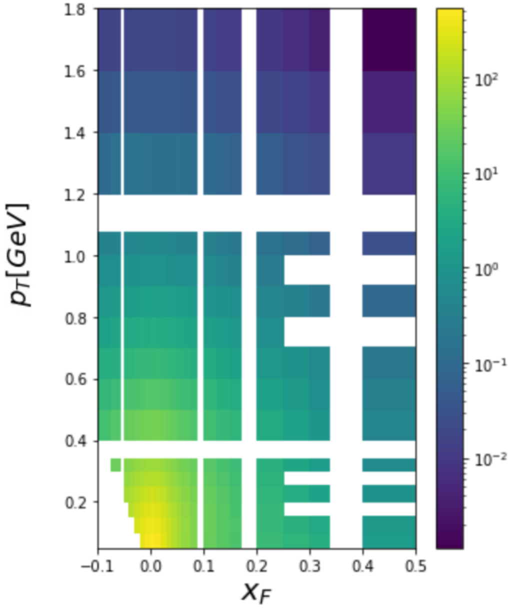
Based on modern proton-carbon data (and pp)

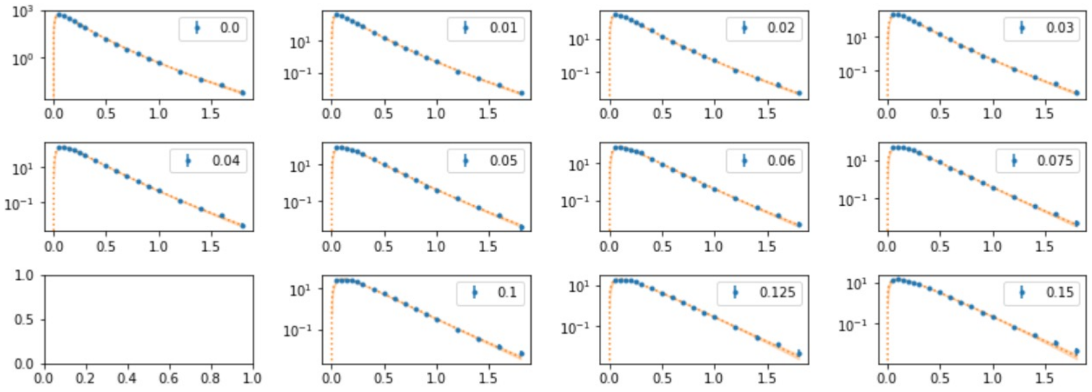
Experiment	beam	$E_{\mathrm{beam}}/\mathrm{GeV}$	Secondaries	Variables
NA49	pC	158	$\pi^{\pm}, \overset{(-)}{\mathrm{p}}, n$	$x_{\mathrm{F}}, p_{\perp}$
NA49	pp	158	$\mathrm{K}^{\pm}$	$x_{\mathrm{F}}, p_{\perp}$
NA61/SHINE	pC	31	$\pi^{\pm}, K^{\pm}, K^0_S, \Lambda$	p, θ
NA61/SHINE	$\pi^{-}\mathrm{C}$	158, 350	$\pi^{\pm}, K^{\pm}, \overset{(-)}{\mathrm{p}}$	$p, p_{\perp}$

AF & M. Huber, PRD 106, 083018 (2022), arXiv:2205.14766

Building the DDM

NA49 proton-carbon @ 158 GeV

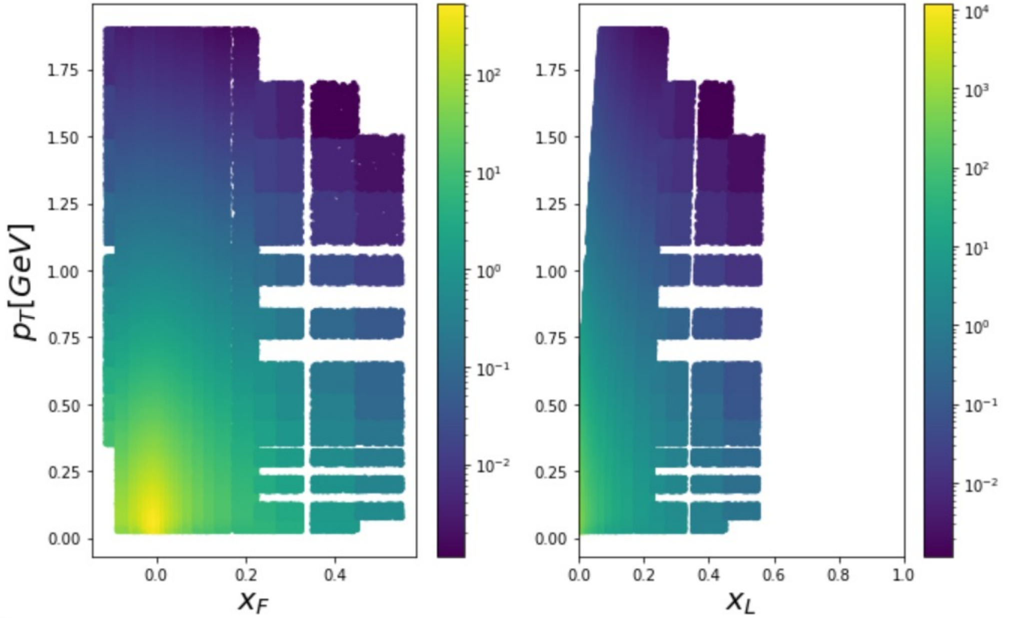




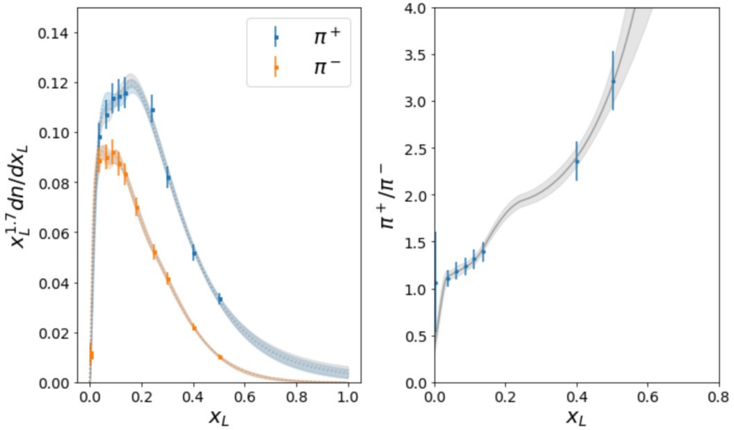
Fit pT in each x_F bin using $\dfrac{dn}{dp_\perp} = a_0 p_\perp^{a_1}\ e^{a_2 p_\perp^{a_3}}$

Sample from
x_F = pz/sqrt(s) and convert into $x_L = E_{secondary}/E_{proj}$

$$x_{Lab} = \frac{E_c}{E_a} = \frac{\gamma\sqrt{m_c^2 + \frac{1}{4}x_F^2 E_{c.m.}^2 + p_{c,T}^{*2}} + \frac{1}{2}\gamma\beta x_F^2 E_{c.m.}}{E_a}$$

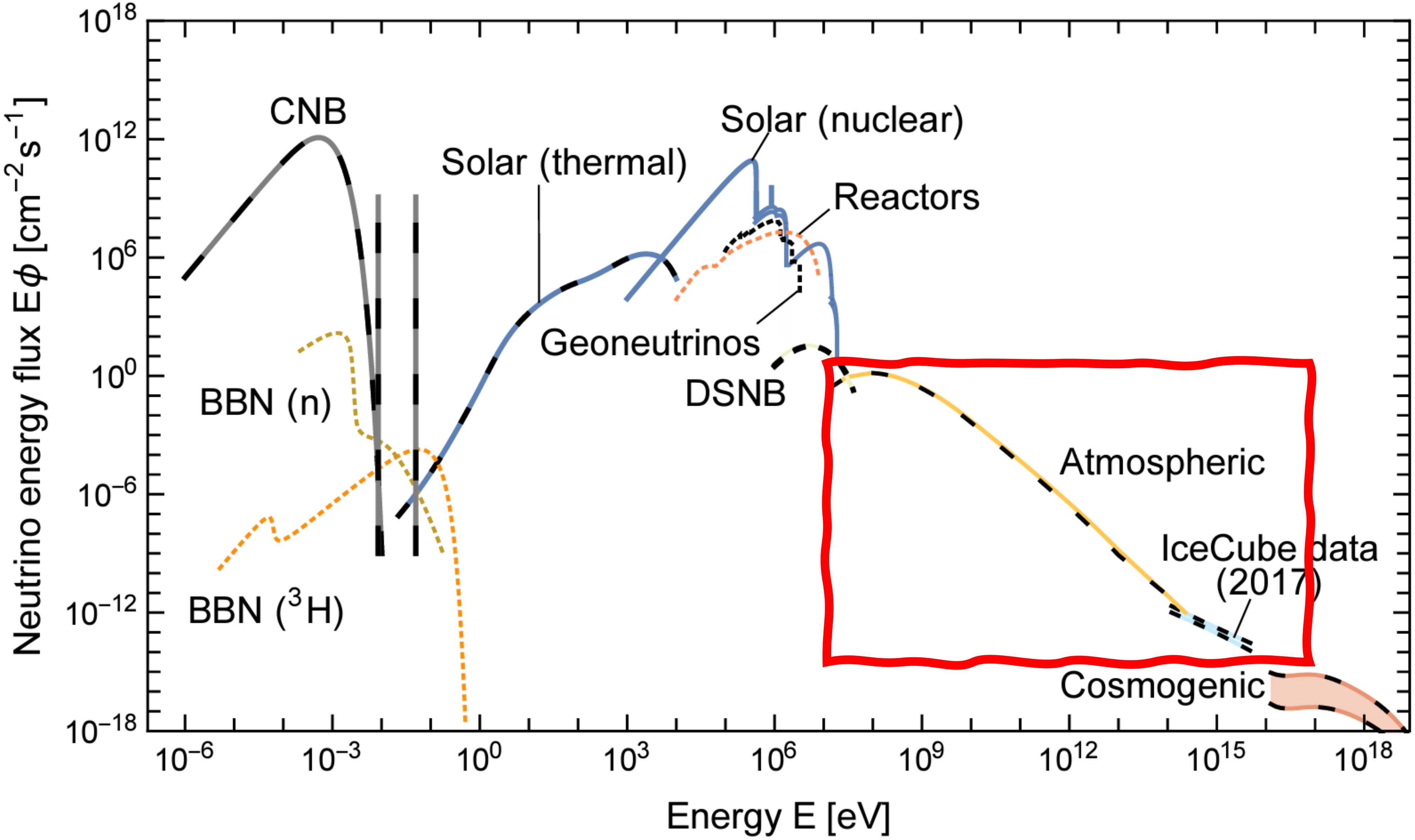


Fit dn/dx_L with splines,
get covariance matrix

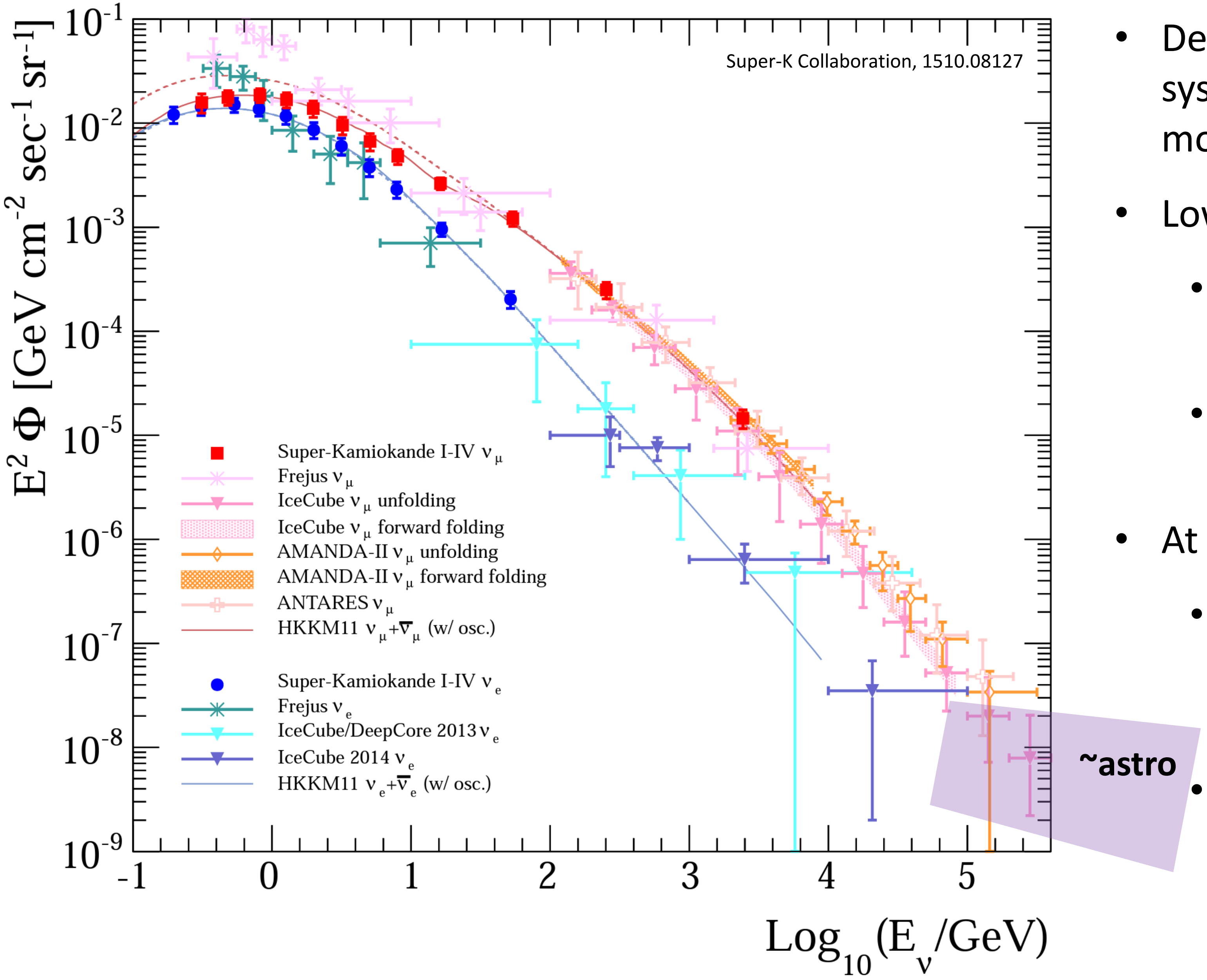


Neutrino spectra at Earth

Vitagliano, Tamborra, Raffelt 2019, 1910.11878



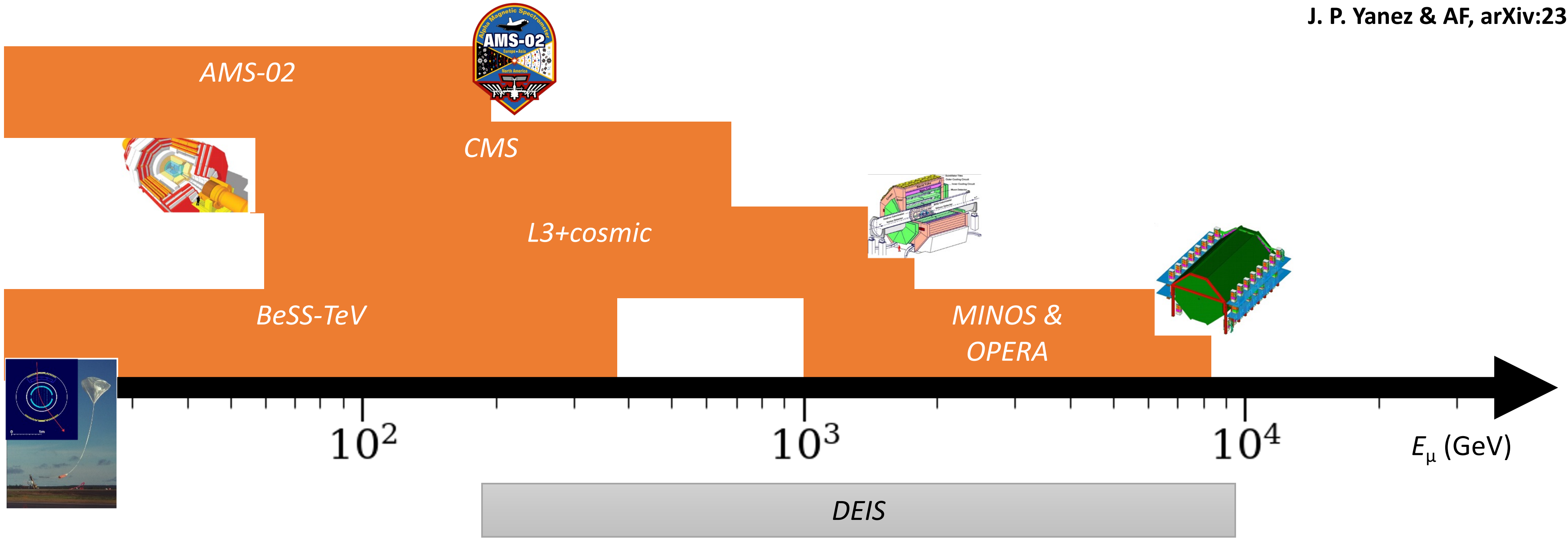
Measurements of atm. neutrinos



- Degeneracy between detector systematics, cross section, assumed flux model and oscillation parameters
- Low energies:
  - Cross section models uncertain -> uncertain norm and spectrum
  - Faint and complex signal -> syst. errors
- At high energies:
  - Muon track from numu charged current not contained withing detectors -> bad energy res.
  - Electron neutrino measurements suffer from lack statistics and neutral current background -> bad stats

`daemonflux`: DAta-drivEn MuOn-calibrated Neutrino Flux

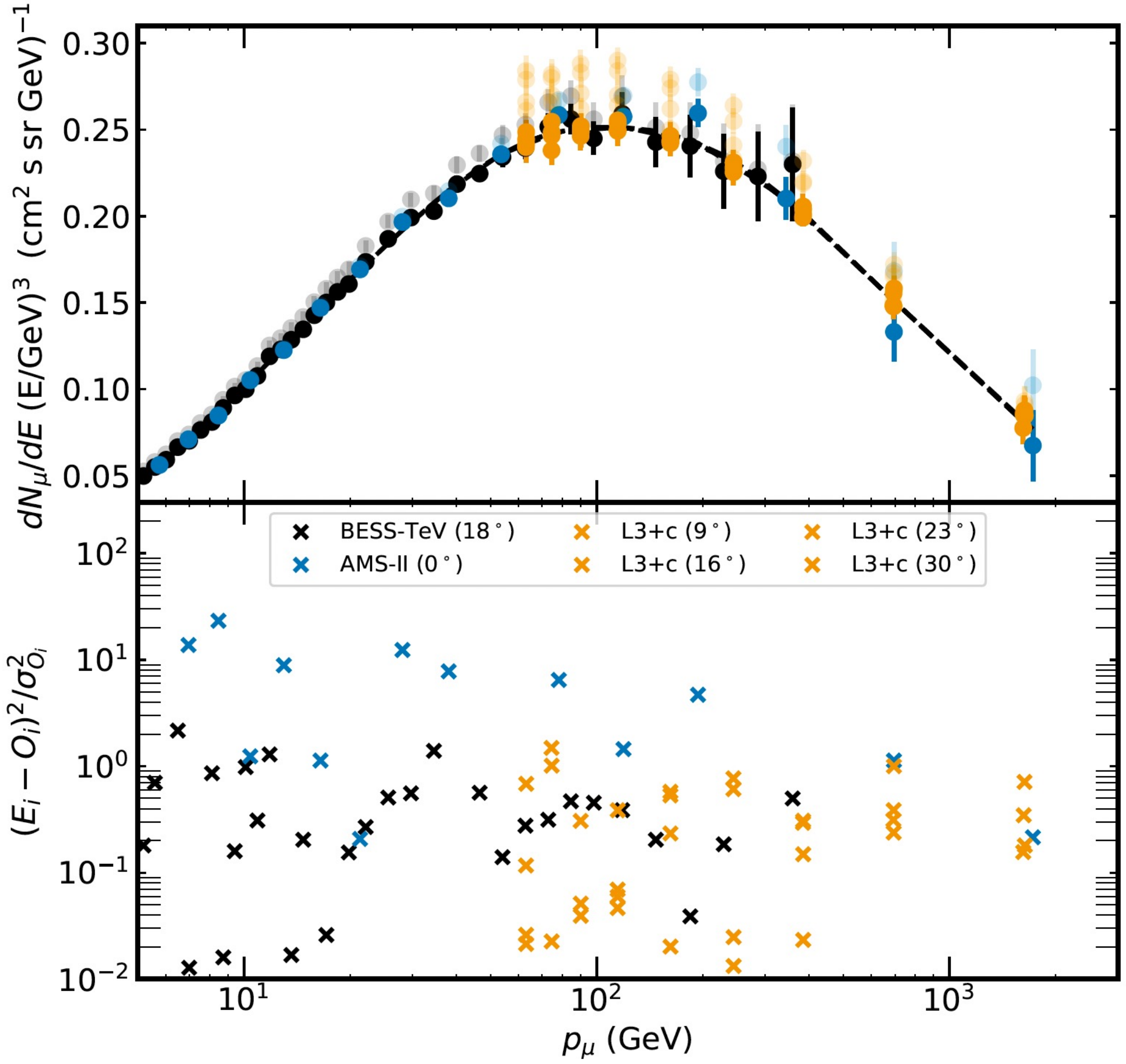
J. P. Yanez & AF, arXiv:2303.00022



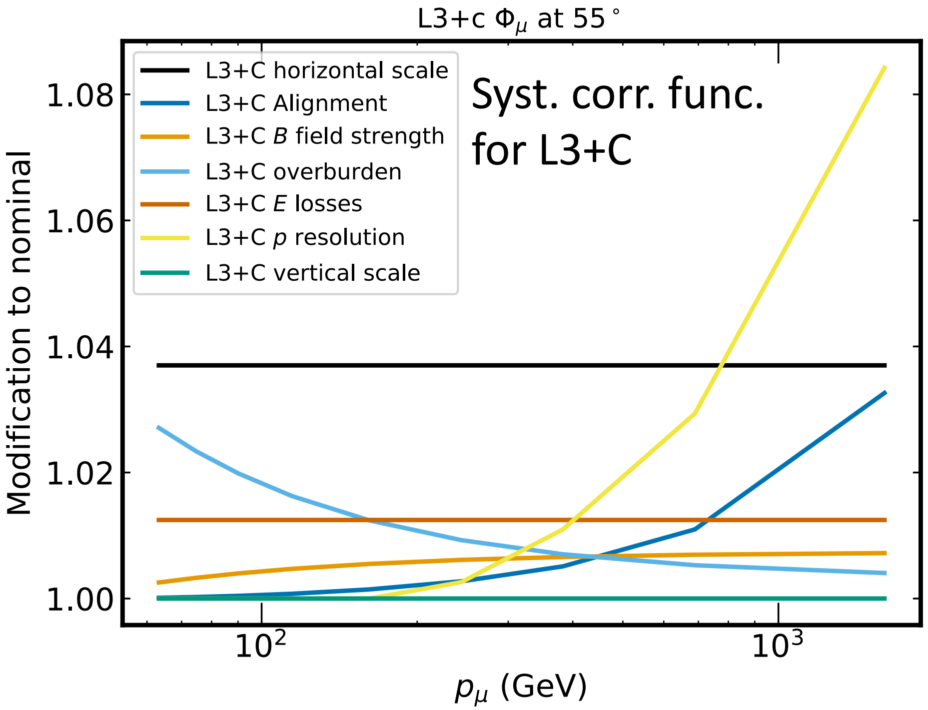
Experiments disclosing systematic uncertainties. Most provide corrrection functions for the data.

Experiment	Energy (GeV)	Measurements	Unit	Systematics	Location	Altitude	Zenith range
BESS-TeV [44]	0.6-400	Φ_μ	p_μ	C	36.2°N, 140.1°W	30 m	0-25.8°
CMS [45]	5-1000	R_{μ^+/μ^-}	p_μ	Q	46.31°N, 6.071°E	420 m	$p\cos\theta_z$
L3+C [46]	20-3000	Φ_μ,R_{μ^+/μ^-}	p_μ	C	46.25°N, 6.02°E	450 m	0-58°
DEIS [47]	5-10000	Φ_μ	p_μ	Q	32.11°N, 34.80°E	5 m	78.1-90°
MUTRON [48]	80-10000	R_{μ^+/μ^-}	p_μ	Q	35.67°N, 139.70°E	5 m	87-90°
MINOS [49]	1000-7000	R_{μ^+/μ^-}	E_μ	C	47.82°N, 92.24°W	5 m	unfolded
OPERA [50]	891-7079	R_{μ^+/μ^-}	E_μ	Q	42.42°N, 13.51°E	5 m	$E\cos\theta^*$

Data compatibility test (no flux model)



- Fit spline in common zenith band with the only requirement that flux has to be smooth. Fit systematic corrections.

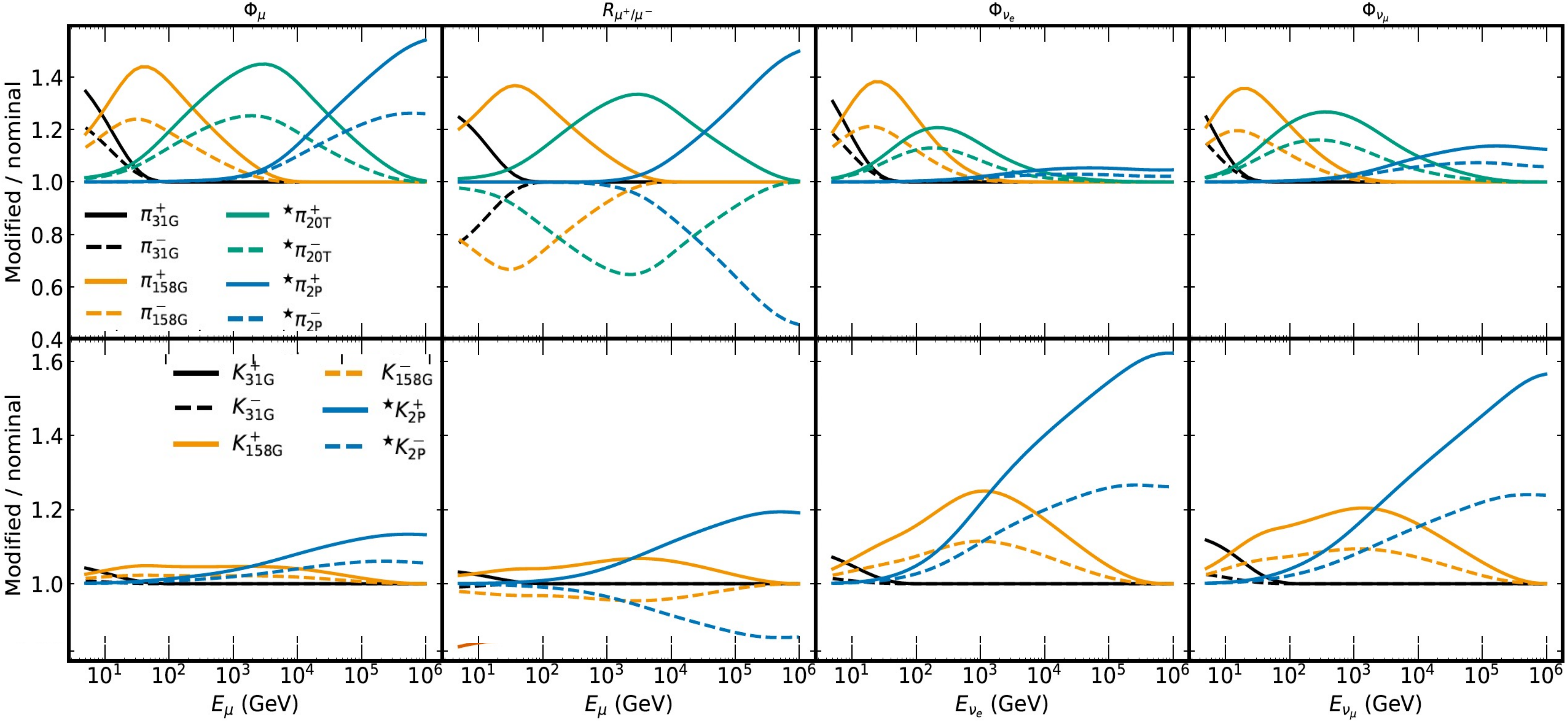


Syst. corr. func. for L3+C

- **Exclude experiment**s, which either are
  - not mutually compatible, or
  - statistically not significant
- or
  - AMS (unpublished PhD thesis)
  - MARS (no competition to BESS)
  - MUTRON (unclear systematics)
  - DEIS (formally OK, but strange induces pulls)

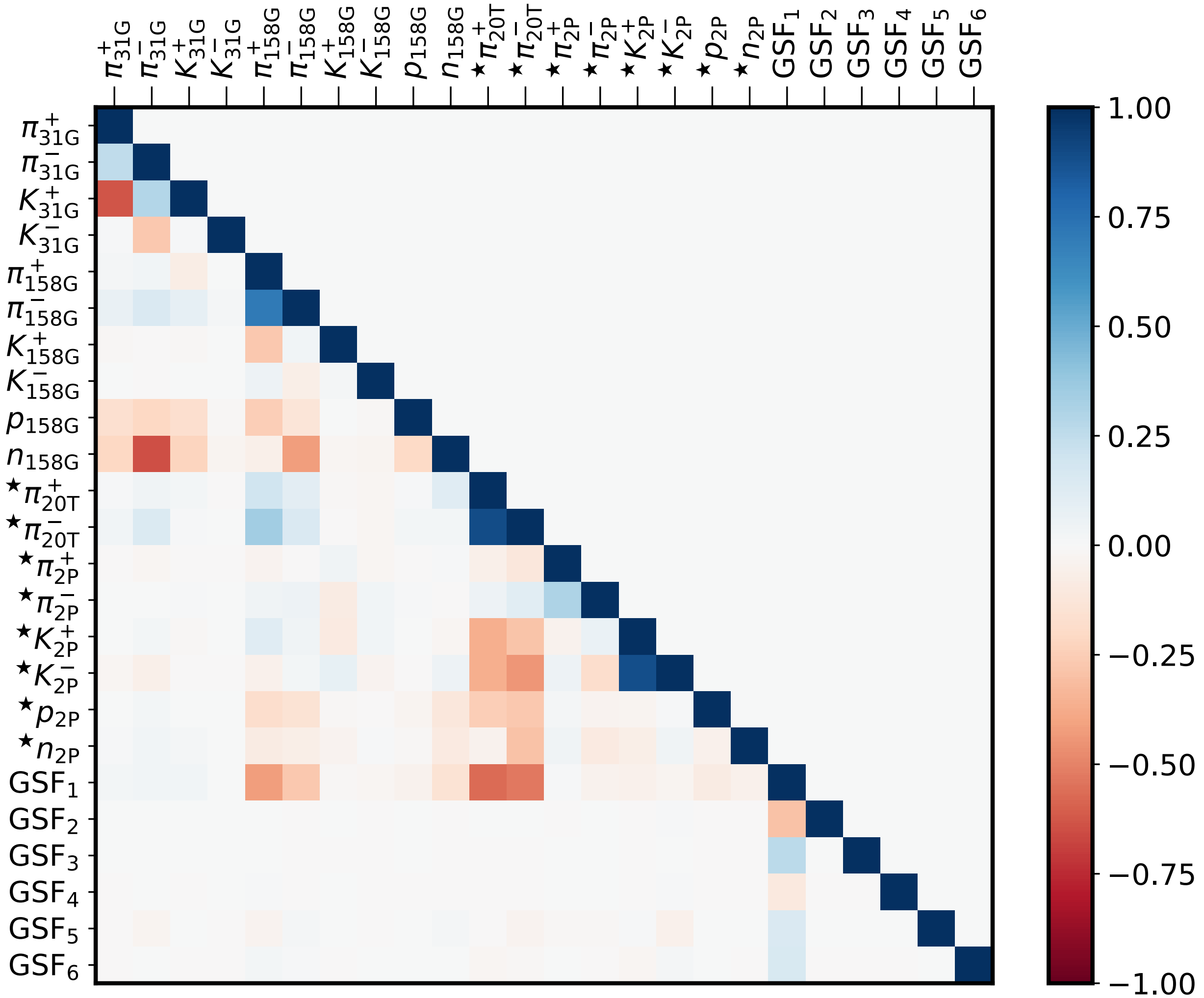
Choice of extrapolation parameters above “DDM energies”

Muon flux **Muon charge ratio** **Muon neutrino flux** **Electron neutrino flux**



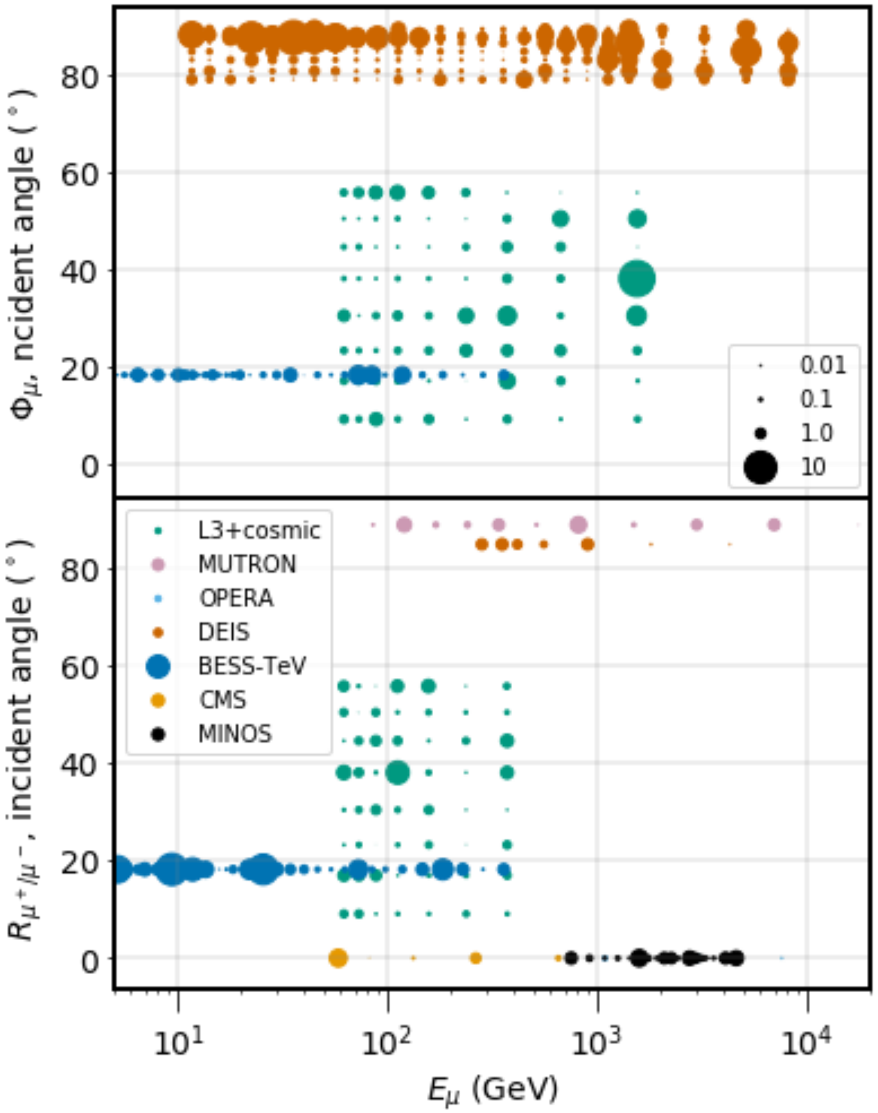
J. P. Yanez & AF, arXiv:2303.00022

Fit quality



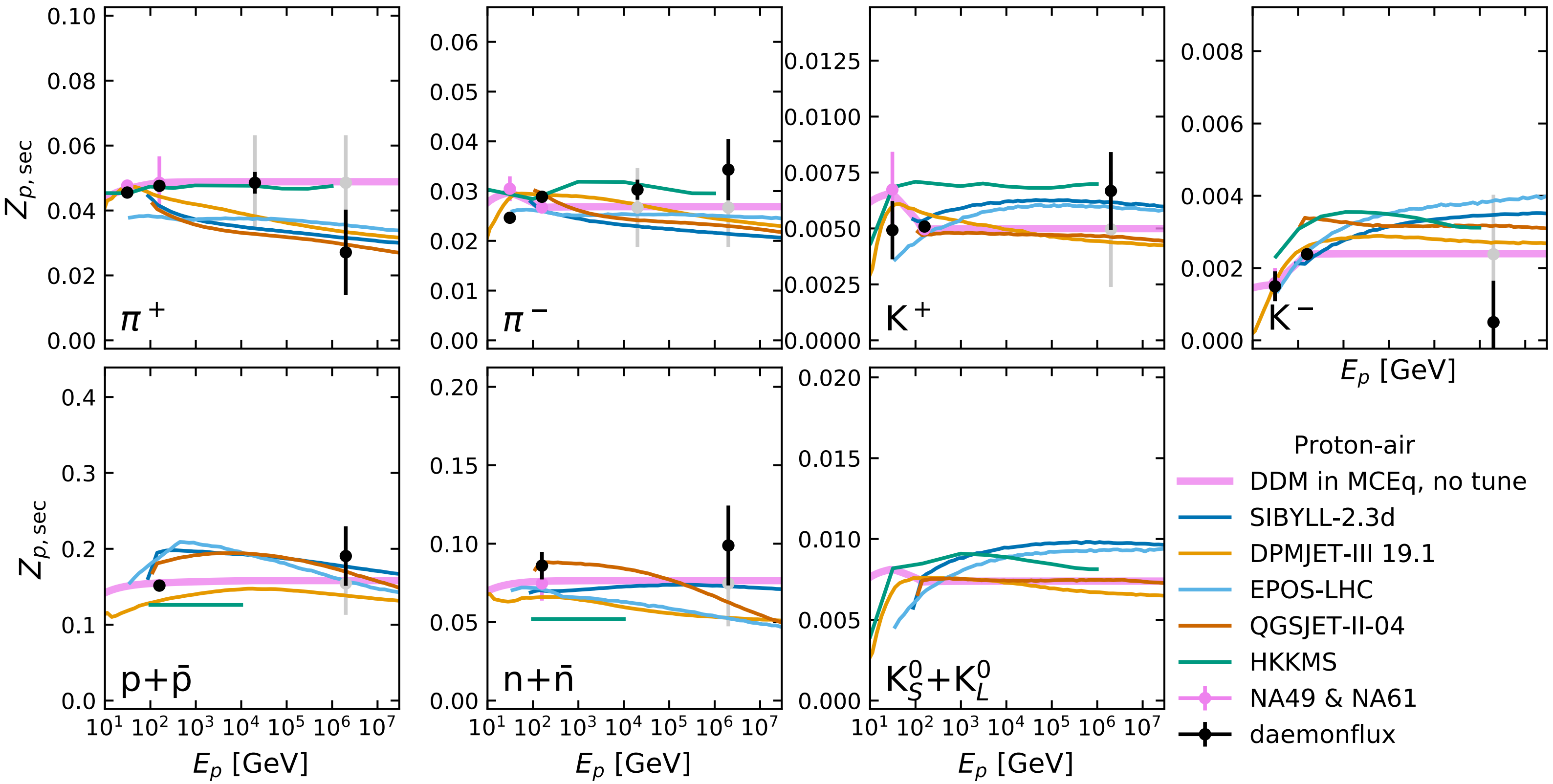
Physics parameter part of the correlation matrix: Total 34 parameters: 18 hadrons + 6 GSF + 10 experimental

J. P. Yanez & AF, arXiv:2303.00022

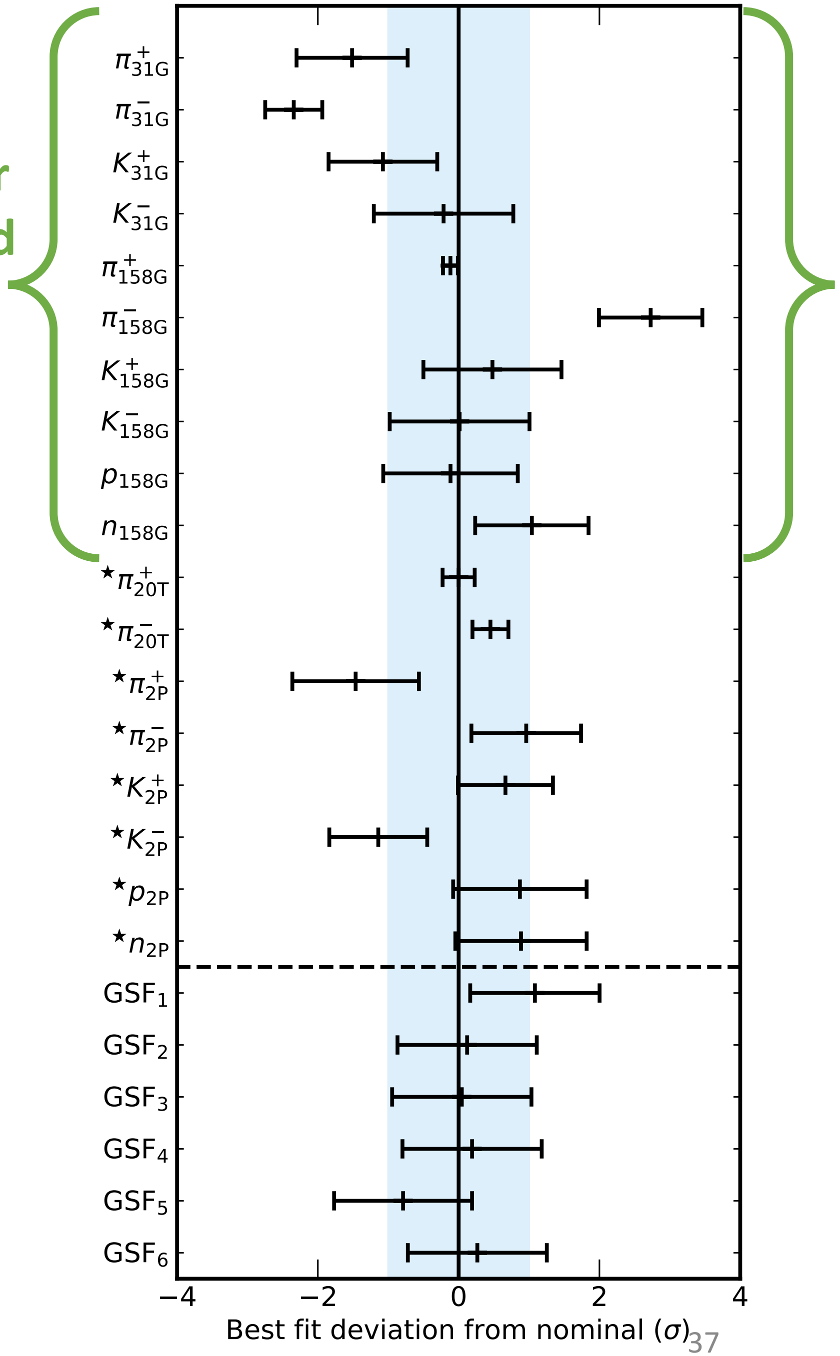


Chi² 199/ 217 dof (approximate)
P-value = 81%

Fitted parameter values



Accelerator constrained



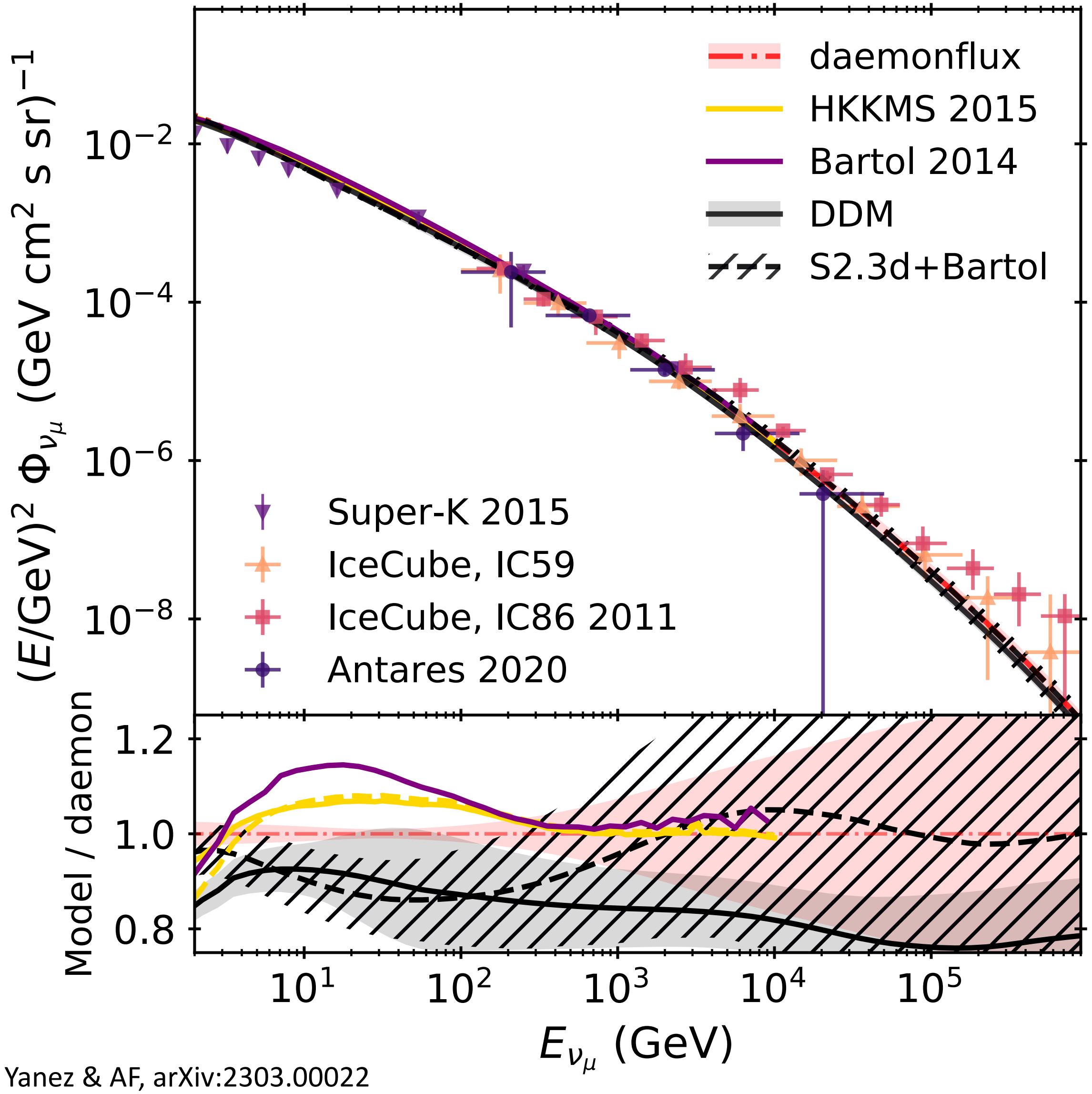
J. P. Yanez & AF, arXiv:2303.00022

Neutrino fluxes

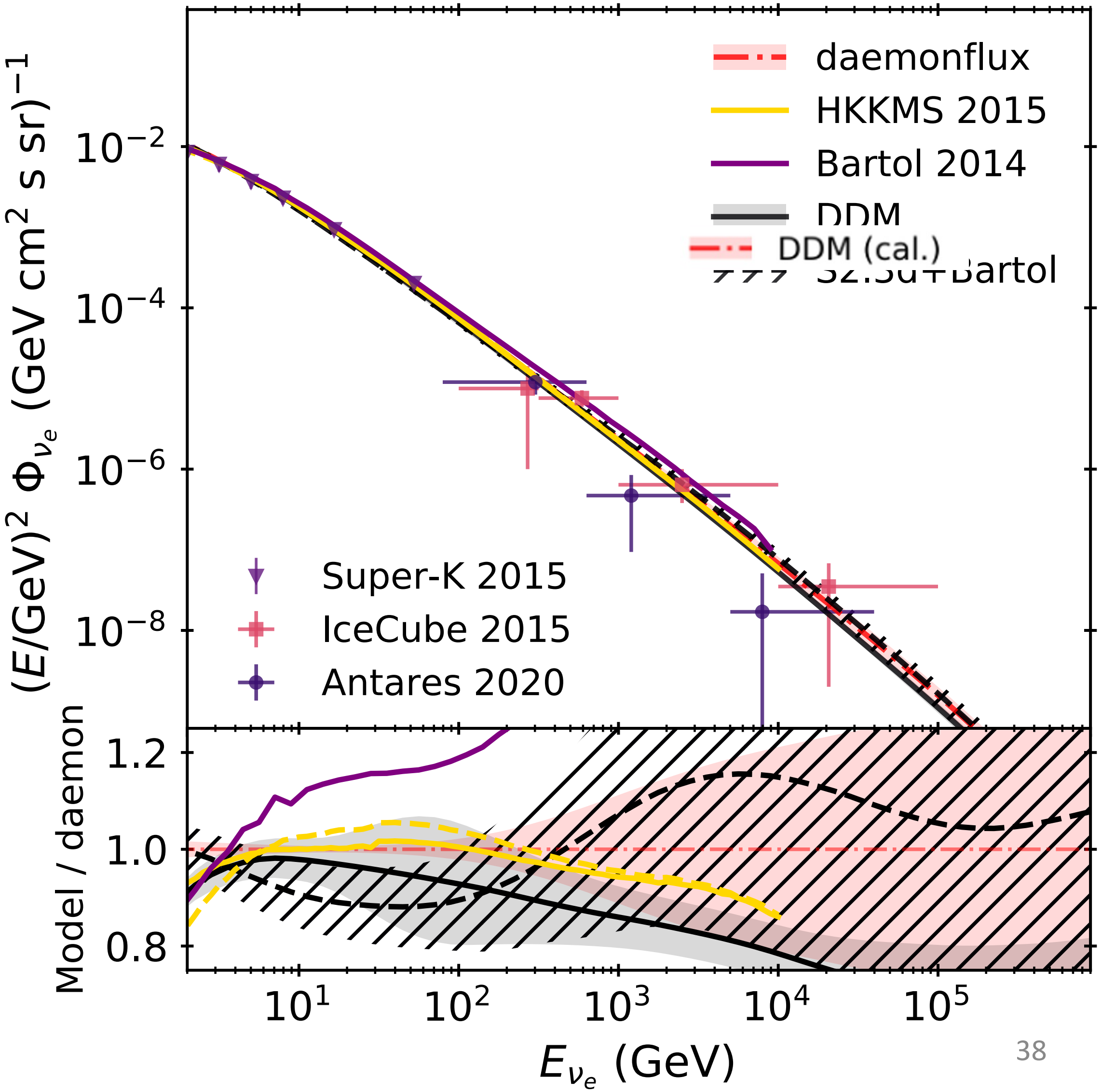
hatched area: uncertainty from

Barr et al. PRD74, 094009 (2006) & AF, Huber PRD (2022)

Muon neutrinos



Electron neutrinos



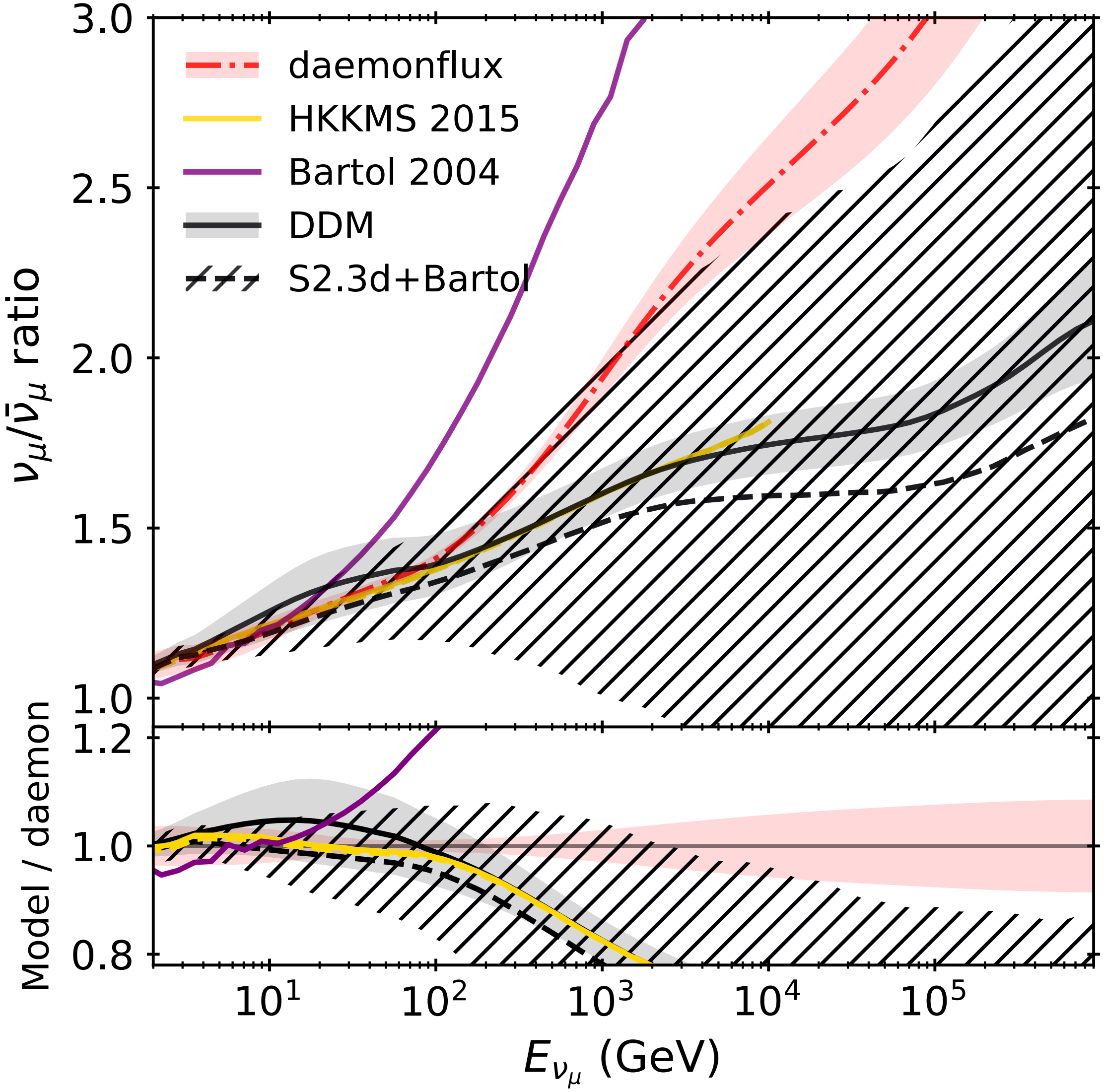
J. P. Yanez & AF, arXiv:2303.00022

Neutrino ratios

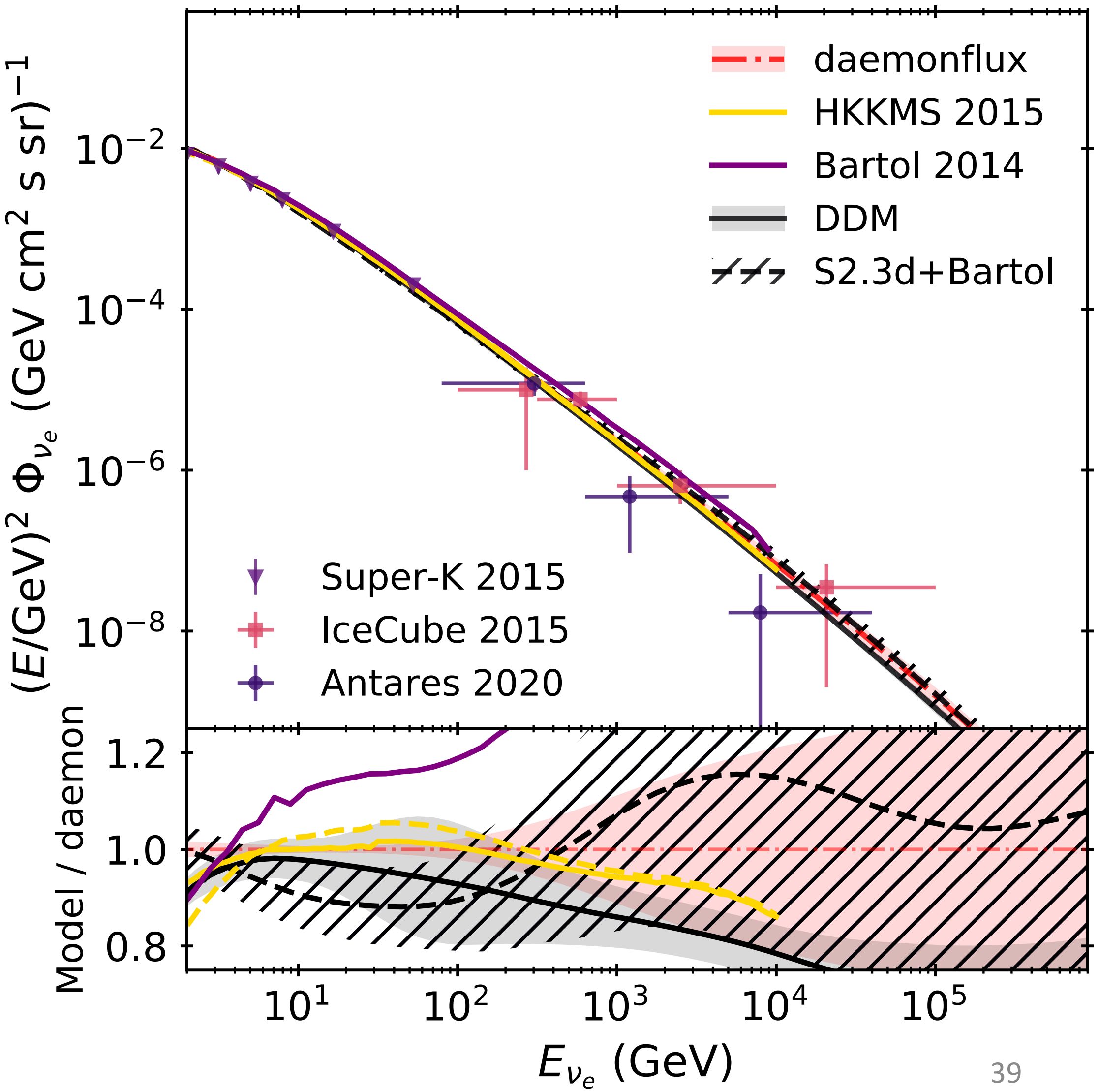
hatched area: uncertainty from

Barr et al. PRD74, 094009 (2006) & AF, Huber PRD (2022)

Numu/numubar ratio

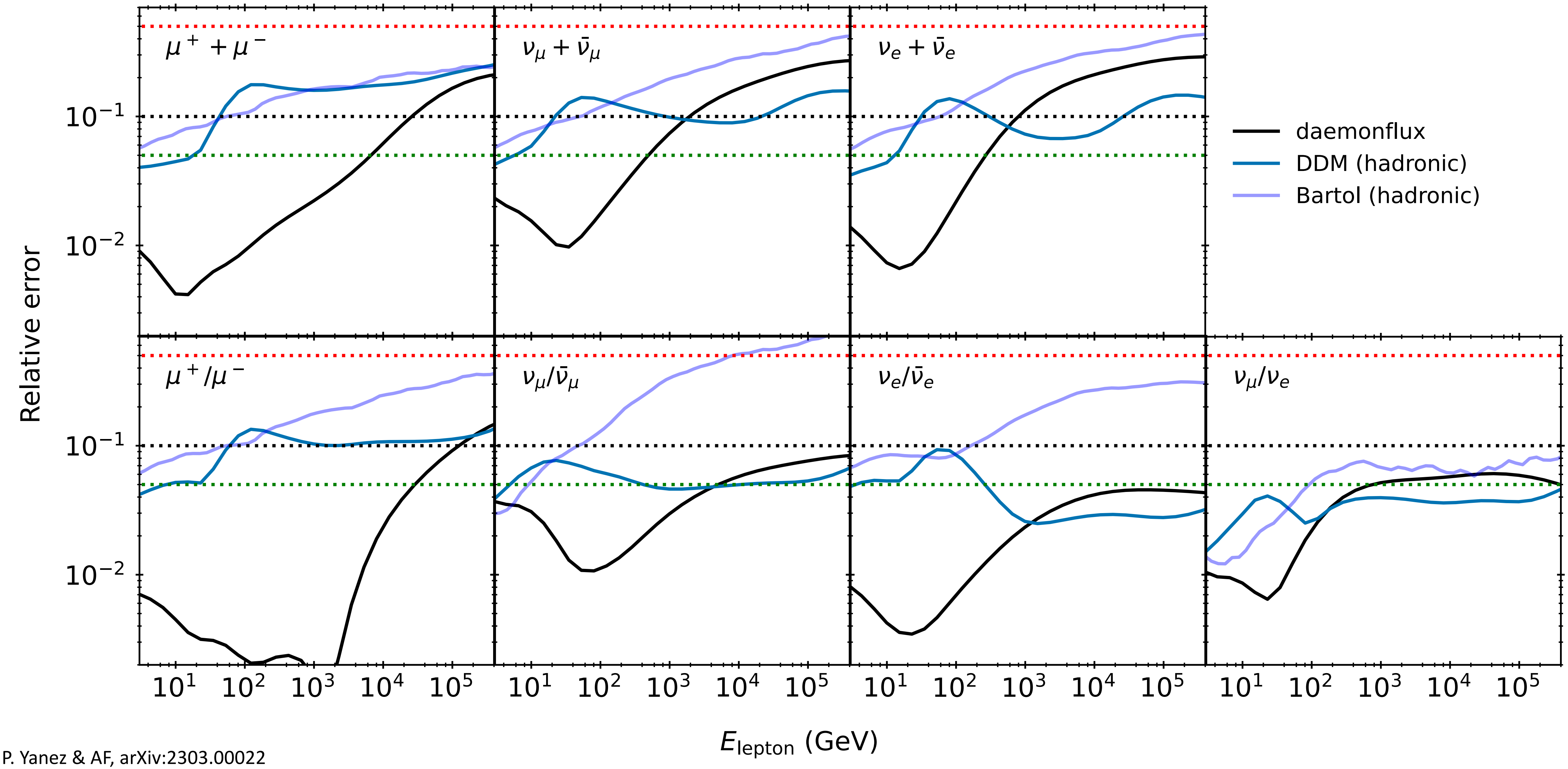


Flavor ratio



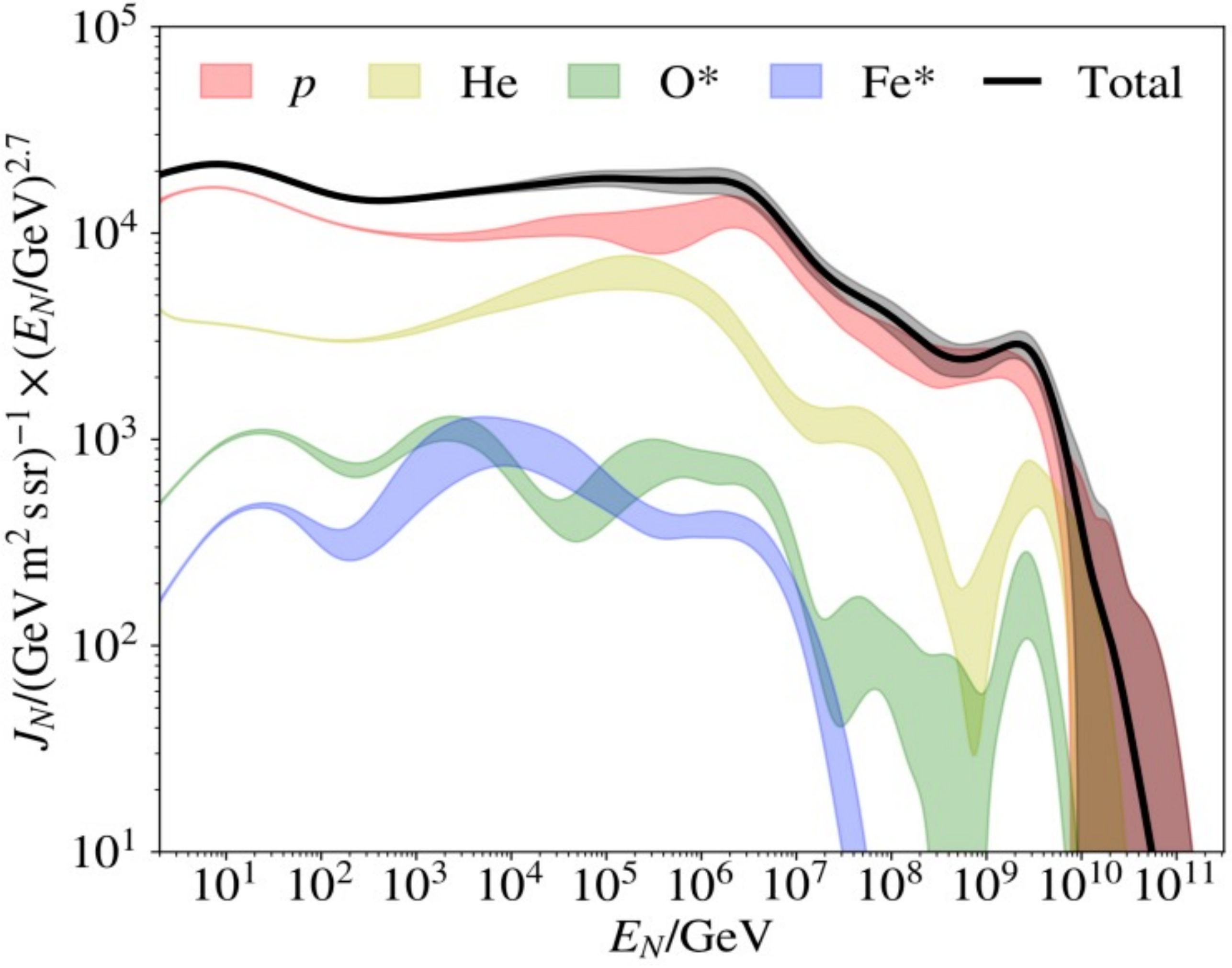
J. P. Yanez & AF, arXiv:2303.00022

Total uncertainty of daemonflux (DDM+GSF+Fit)



J. P. Yanez & AF, arXiv:2303.00022

The Global Spline Fit – nucleon fluxes (MCEq input)



- Most contribution from proton and helium flux
- Correlations between H and He affect
  - CR neutron fraction
  - Muon charge ratio
  - Neutrino/Antineutrino ratio

→ Need to model two correlated components

→ technically ~80 parameters

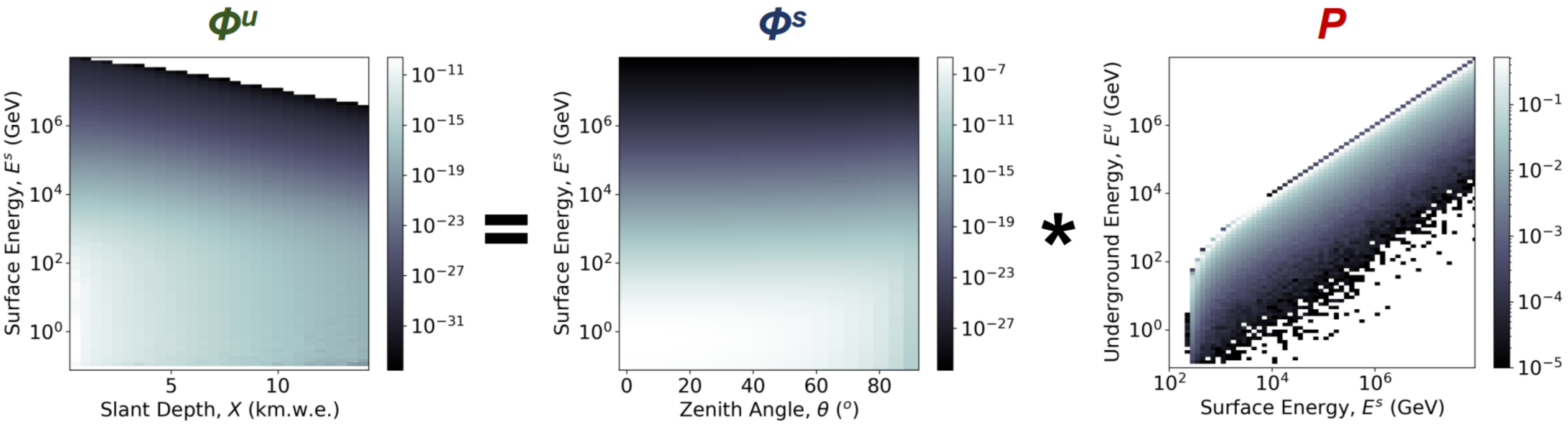
AF, Dembinski, Engel, Riehn, Gaisser, Stanev ICRC 2017

MUTE (Muon inTnsity codE): fast convolutions

AF, **W. Woodley**, M.-C. Piro, *ApJ* **928** 27 (2022)
W. Woodley, TeVPa 2022 and Woodley, AF, Piro in prep.

https://github.com/wjwoodley/mute

$$\Phi^u(E_j^u, X_k, \theta_k) = \sum_i \Phi^s(E_i^s, \theta_k) P(E_i^s, E_j^u, X_k) \left(\frac{\Delta E_i^s}{\Delta E_j^u}\right)$$



MUTE
Underground Fluxes

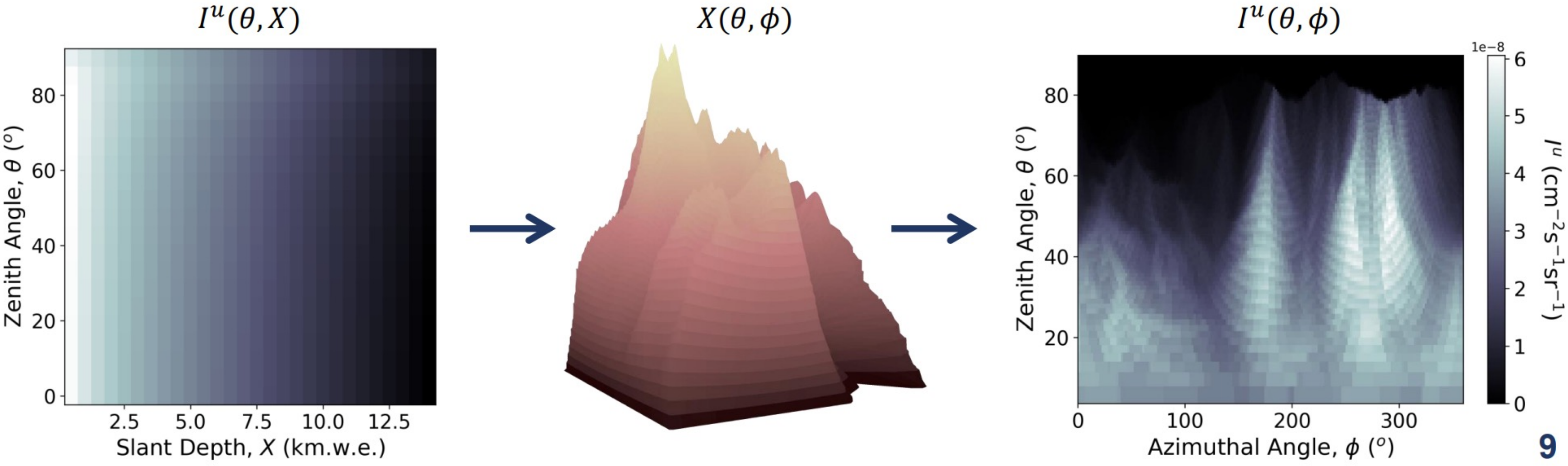
MCEq
Surface Fluxes

PROPOSAL
Transfer Tensor

MUTE (Muon inTnsity codE): Muon flux for labs under mountains

https://github.com/wjwoodley/mute

$$\Phi^u = \iint_\Omega I^u(X(\theta,\phi),\theta)\mathrm{d}\Omega.$$



W. Woodley, TeVPa 2022